BENG 186B Principles of Bioinstrumentation

Week 4 Review

Exercises

Selections from:

2015 Homework 3
2015 Homework 4
1. **Nernst Potentials:** Consider the following circuit model for a cell. 

\[ R_K = 2.7 \text{ k}\Omega, \; R_{Na} = 30 \text{ k}\Omega, \; R_{Cl} = 3.3 \text{ k}\Omega. \]

(a) With the intracellular and extracellular concentrations given in the table below, calculate the Nernst Potential for each of the ionic species: \( V_K \), \( V_{Na} \), and \( V_{Cl} \).

<table>
<thead>
<tr>
<th>Ionic Species</th>
<th>Intracellular concentration</th>
<th>Extracellular concentration</th>
</tr>
</thead>
<tbody>
<tr>
<td>( K^+ )</td>
<td>397 mM</td>
<td>20 mM</td>
</tr>
<tr>
<td>( Na^+ )</td>
<td>50 mM</td>
<td>437 mM</td>
</tr>
<tr>
<td>( Cl^- )</td>
<td>40 mM</td>
<td>556 mM</td>
</tr>
</tbody>
</table>

(b) Using the circuit model with Nernst potentials \( V_K \), \( V_{Na} \), and \( V_{Cl} \) and the resistances given above, find the membrane potential \( V_M \) at steady-state.

*Hint: At DC steady-state, any capacitance reduces to an open circuit connection.*

(c) Now find the equilibrium resting potential \( V_M \) using the Goldman-Hodgkin-Katz equation. Compare the two values of the membrane potential. Which value is more reasonable for a typical resting potential of a cell?

*Hint: Membrane conductance (the reciprocal of membrane resistance) for any ion type is directly proportional to membrane permeability for that ion type.*

(d) For a membrane capacitance \( C_m = 1 \mu\text{F} \), find the time constant for the membrane potential \( V_M \) of the cell to recover from a transient and settle to its steady-state value.

2. **Electroporation:** Often it is necessary to insert genetic material into a cell, crossing its membrane. Electroporation is one means to open the membrane for insertion through the application of a high voltage. Here we study single cell electroporation, by injection of currents \( +I \) and \( -I \) through two nearby electrodes into the extracellular space, as shown in the figure below. The voltage across A and B, on both sides of the the cell, should not exceed 400 mV, otherwise the cell may die. The conductivity of the extracellular medium is \( \sigma = 1 \Omega^{-1}\text{m}^{-1} \),...
the inter-electrode distance is $X_i = 250 \, \mu m$, the cell is midway between the electrodes centered at distance $X_m = 125 \, \mu m$ from either electrode, and the cell diameter is $d = 20 \, \mu m$. Determine the maximum amplitude of the current $I$ you can safely inject.

*Hint:* Express the voltage $V_A - V_B$ across the cell as a difference between two biopotentials generated by the same current dipole.

3. **Intracellular electronic recording:** In electrophysiology there are several techniques used to measure various aspects of electric activity in single cells. One such technique is the current clamp, which injects a current into the cell and measures the resulting membrane voltage of the cell. Based on the design specifications below design a circuit which accomplishes a current clamp for intracellular voltage recording.

   (a) You are given a variable current source that is set to the desired current clamp value.
   
   (b) The reference electrode is located in the extracellular space, and the recording electrode reaches inside the cell (through a glass pipette penetrating the cell membrane).
   
   (c) The signal you are trying to measure at the recording electrode is in the range of $-80 \, mV$ to $40 \, mV$.
   
   (d) Amplify the signal so an external voltmeter measures $1.2 \, V$ at $40 \, mV$ and $0 \, V$ at $-80 \, mV$. The output voltage should vary linearly with input voltage. The input impedance of your amplifier should be near-infinite.
   
   (e) $60 \, Hz$ line noise present on the electrode wires should be reduced in the amplifier output.
   
   (f) The frequency range of interest in the voltage signal is $100 \, Hz$ to $10 \, kHz$.

*Bonus:* Design a voltage clamp circuit to the same above specifications for the current clamp, except the signal you are measuring is now current into the recording electrode for a fixed voltage across the electrodes, and the current signal ranges between $-250 \, nA$ to $+250 \, nA$, where the corresponding output voltage should range from $-1 \, V$ to $1 \, V$.  

2
1. Cardiac vectors:

(a) Given the following ECG of Lead I and knowing that the amplitude of the R-wave in the Lead II ECG is 10 mV, determine the magnitude and direction of the cardiac vector $\mathbf{M}$, assuming it remains constant throughout the ECG. The length of each of the leads is 1 m.

(b) Neatly sketch the ECG for Lead II and Lead III. On each of the graphs, label the axes and show the values of ECG amplitude at the P, R, and T waves.
2. **Electrode model**: Consider the following skin–electrode model below:

(a) Find the expression for the impedance of the circuit when the subject is not sweating, that is, disregarding the sweat glands and ducts contribution.

(b) Find the expression for the impedance of the circuit when the subject is sweating, that is, including the sweat glands and ducts contribution.

(c) Using the following parameter values sketch (or plot) the magnitude of the impedance as a function of frequency, from 0.1Hz to 100Hz on a log-log scale. Make sure to properly label your plot with values and units.

\[
\begin{align*}
E_{hc} &= 200 \text{ mV} \quad R_s = 1 \text{ k}\Omega \quad C_d = 1 \text{ pF} \quad C_e = 10 \text{ pF} \quad C_p = 0 \\
E_{sc} &= 430 \text{ mV} \quad R_u = 100 \text{ k}\Omega \quad R_d = 1 \text{ M}\Omega \quad R_e = 10 \text{ M}\Omega \quad R_p = \infty
\end{align*}
\]
3. **ECG amplifiers**: Consider the four-electrode frontal lead ECG bioamplifier system below:

![Diagram of ECG amplifiers](image)

(a) Derive the lead voltage outputs $VR$, $VL$, and $VF$ in terms of the electrode voltages $RA$, $LA$, and $LL$, respectively. What are the voltage gains on each of these leads?

(b) Derive the driven right leg voltage output $RL$ in terms of the common mode of $RA$, $LA$, and $LL$. What is the common mode voltage gain driving the right leg?

(c) Find the effective resistance from body to ground in terms of the RL electrode impedance $R_{RL}$ and the resistances of the circuit.

(d) What purpose do resistors $R_i$ and $R_o$ serve in this circuit?

(e) **BONUS**: Using additional opamps and resistances, augment your circuit to produce six amplified leads I, II, III, aVR, aVL, and aVF from the $VR$, $VL$ and $VF$ outputs. Express the total voltage gain of the leads in terms of the resistances.
4. **Design problem:** Your task is to design part of an internal (or implanted) automatic defibrillator (IAD) operating from a single 1.2 V battery. Your design has three parts.

(a) The IAD has three electrodes (ground and two lead I electrodes) available for the purposes of obtaining electrocardiograms. Design an amplifier so the IAD microcontroller can observe heart activity.
   - Assume that the electrodes include protection circuitry for overvoltage protection during IAD electric shock delivery.
   - This amplifier should have a gain of at least 1000.
   - The amplifier should filter out signals above 1 kHz.
   - The amplifier should use active grounding. The ground should not pass more than 5 μA and should reduce the common mode signal by at least a factor of 1000.

(b) The IAD defibrillator is configured to perform cardioversion when needed by delivering a high-energy electrical shock to the heart. It is critical that such shock never be delivered during a T wave, or ventricular tachycardia may be induced which may be fatal. Design a circuit to block the IAD from delivering any shock for a duration of 400 ms after every R wave in the ECG, spanning the T wave with sufficient timing margin. The circuit should generate a digital control signal that goes high (1.2 V) for the duration of the block.

(c) The IAD includes an ultrasonic transmitter to notify an external monitor whenever a shock is being delivered. Design a circuit that generates a 500 kHz 1.2 V square wave driving an ultrasonic speaker whenever the IAD cardioverter is active. The cardioverter goes active when it is driven to 1.2V by the IAD microcontroller.