BENG 221 Mathematical Methods in Bioengineering

Fall 2014

Midterm

NAME: __________________________________________

• Open book, open notes.
• 80 minutes limit (end of class).
• No communication other than with instructor and TAs.
• No computers or internet, except for access to posted class materials.
**Problem 1**  (30 points): Short answer problems. Provide brief explanations (no lengthy derivations!) for each problem.

1. (5 points): A random walk process with random step size of standard deviation $\Delta x$ and step time interval $\Delta t$ is characterized with a diffusivity $D$. How does the diffusivity $D$ change as the random step size increases by a factor two (i.e., $\Delta x$ becomes $2\Delta x$)?

2. (5 points): Conversely, how does the diffusivity $D$ change as the step time interval increases by a factor two (i.e., $\Delta t$ becomes $2\Delta t$)?

3. (10 points): Find the radial frequency $\omega$ of wave oscillations of a free electron with energy $E$.

4. (10 points): Find the maximum time step $\Delta t$ beyond which Euler numerical integration of the ODE $dx/dt = -x/\tau$ becomes unstable (i.e., gives unbounded results).
Problem 2  (25 points): Consider an electrically excitable cell as shown below. The cell has membrane capacitance $C_m$ and leak conductance $g_i$. The extracellular space has leak conductance $g_e$. Both intracellular and extracellular potentials are initially zero. At time zero, a constant current electrode current $I_{elect}(t) = I_0$ is injected into the extracellular space.

1. (5 points): Write the differential equations and initial conditions governing the dynamics of the intracellular and extracellular potentials, $v_i(t)$ and $v_e(t)$.

2. (15 points): Find the intracellular potential $v_i(t)$ over time. You may use Laplace or any method of your choice.
3. (5 points): Find the minimum level of electrode current $I_0$ to be injected in order for the intracellular potential $v_i(t)$ to reach the threshold $V_{th}$ at which the cell generates an action potential. At what time does the action potential happen? Does the duration of the electrode current matter, and how?
Problem 3  (45 points): An athlete initially at rest starts to exercise. The body is covered with thermally insulating material. Underneath the skin \((x = 0)\) is muscle tissue of thickness \(L\), interfacing on the other end \((x = L)\) with vasculature. The thermal conductivity of the muscle tissue is \(K_0\), and the vasculature conducts heat to maintain the tissue interface at a constant temperature \(T_0\). Specific heat of the muscle tissue is \(c\), and mass density is \(\rho\). Once starting to exercise \((t \geq 0)\), the athlete burns calories (Joules) uniformly in the muscle tissue at constant rate, with heat generation \(Q(x, t) = Q_0\).

1. (5 points): Write the partial differential equation governing temperature \(u(x, t)\) in the muscle tissue. Express initial and boundary conditions.

2. (10 points): Solve for a particular solution \(u_p(x)\) for the temperature in the tissue at steady state.
3. (5 points): Write the homogeneous problem, with homogeneous partial differential equation and boundary conditions. You may combine constants into a thermal diffusivity $D$.

4. (10 points): Find the eigenmode decomposition for the general solution of the homogeneous problem.
5. (15 points): Solve for the temperature in the tissue over time \( u(x, t) \) from initial conditions.