BENG 186B Principles of Bioinstrumentation

Week 7 Review

Solutions

Selections from:

2015 Homework 5
2015 Homework 6
1. One of the major problems during dialysis after bleeding out is the introduction of air bubbles as those can lead to embolism, which can result in cardiac arrest depending on where the embolism lodges. A safety check within the machine is to incorporate a bubble detector to warn clinicians on staff if bubbles have formed. As an engineer testing such a detection system, you receive the following signals from the system driven with a signal generator to mechanically stimulate test samples under two conditions: with and without bubbles. In each case the mechanical actuation by the signal generator couples to a membrane with compliance $C_d = 0.001 \text{ Pa}^{-1}$ in fluidic contact with the test sample.

![Graph showing normalized voltage over time for signals A and B.](image)

(a) Assuming that the amplitude of the signal generator voltage is kept the same in both tests with and without bubbles, and the signal generator is tuned to operate at the natural frequency in each case, determine which of the signals A and B correspond to the test with or without bubbles respectively. Explain why.

(b) Determine the compliance of bubbles ($C_b$) with the parameters from the given plot. Assuming that $C_b$ is directly proportional to the total volume of air inside the test system, explain how you can use the bubble detector signal to calculate the bubble density.
1. Bubble detector:
   a. Frequency rationale:
      \[ f_n = \frac{\omega_n}{2\pi}, \text{ where } \omega_n = \frac{1}{\sqrt{LC}} \]
      \[ C_{no\ bub} = C_0, \text{ and } C_{bub} = C_0 + C_b \]
      Hence, \( C_{no\ bub} < C_{bub} \)
      Therefore, \( \omega_{no\ bub} > \omega_{bub} \)
      From the plot, \( \omega_{nA} < \omega_{nB} \)
      Therefore \( \omega_{nA} = \omega_{bub} \) and \( \omega_{nB} = \omega_{no\ bub} \)
   
   Amplitude rationale:
   
   \[ \left| \frac{V_{out}}{V_{source}} \right| = \frac{1}{2\xi}, \text{ where } \xi = \frac{R}{2} \sqrt{\frac{C}{L}} \text{, } C_{no\ bub} = C_0, \text{ and } C_{bub} = C_0 + C_b \]
   
   Hence, \( C_{no\ bub} < C_{bub} \) and \( \xi_{no\ bub} < \xi_{bub} \)
   
   Therefore \( \left| \frac{V_{out}}{V_{source}} \right|_{no\ bub} > \left| \frac{V_{out}}{V_{source}} \right|_{bub} \)

   From the plot \( \left| \frac{V_{out}}{V_{source}} \right|_A < \left| \frac{V_{out}}{V_{source}} \right|_B \)

   Therefore \( \left| \frac{V_{out}}{V_{source}} \right|_A = \left| \frac{V_{out}}{V_{source}} \right|_{bub} \) and \( \left| \frac{V_{out}}{V_{source}} \right|_B = \left| \frac{V_{out}}{V_{source}} \right|_{no\ bub} \)

   b. \( \left| \frac{V_{out}}{V_{source}} \right| = \frac{1}{2\xi} = \frac{1}{R} \sqrt{\frac{L}{C}} \)

   Bubbles: \( \left| \frac{V_{out}}{V_{source}} \right| = \frac{1}{R} \sqrt{\frac{L}{C_0 + C_b}} \)

   No bubbles: \( \frac{1.5V_{out}}{V_{source}} = \frac{1}{R} \sqrt{\frac{L}{C_0}} \)

   Divide and cancel terms, \( \frac{1.5}{1} = \sqrt{\frac{C_0}{C_0 + C_b}}, \text{ so } C_b = 0.00125 \text{ Pa}^{-1} \)

   Substitute \( C_0 = 0.001 \text{ Pa}^{-1} \)

   Relationship with bubble density

   Divide and cancel terms, \( \frac{V_{outA}}{V_{outB}} = \sqrt{\frac{C_0}{C_0 + C_b}}, \text{ so } C_b = C_0 \left( \left( \frac{V_{outB}}{V_{outA}} \right)^2 - 1 \right) \)

   Assuming \( C_b = k [volume_{air}] \) where \( k \) is constant and since \( C_0 \) is also constant, then \( [volume_{air}] \) depends only on \( \left( \frac{V_{outB}}{V_{outA}} \right)^2 \) and assuming that the volume of blood within the dialysis machine is fixed, the bubble density can be calculated.
2. For many biological problems, cell sorting is an important task of differentiating and categorizing different cell types within a heterogeneous mixture. Utilizing the Hall Effect phenomena, you design a system where you can separate cells of interest by using charged antibodies that attach specifically to the antigens of your cells. Since you have two cell types to separate, you design negatively and positively charged antibodies.

Due to the separation of charges, the tube through which the cells flow acts like a parallel plate capacitor with a distance of 5 mm. The cells are flowing at 100 mL/s through the tube with a relative permittivity of 80. You induce the Hall Effect after generating a magnetic field of 0.25 T.

(a) Draw the system with the corresponding vectors for the generated magnetic field, flow of cells, and electric field.

(b) Assuming each cell to have a valance of either +1 or −1, calculate the concentration of cells of either type covering the walls of the tube.

3. You have a device that can measure blood velocity using ultrasonic pulses. This requires the user to position the source and receiver correctly.

(a) Given the following different setups, determine the expected frequency shifts when using an ultrasound sound system to measure the blood velocity.

i. $S \rightarrow R$

\[
\theta_s, \theta_r
\]

\[
v
\]

ii. $S = R$

\[
\theta_s = \theta_r
\]

\[
v
\]

iii. $S$

\[
\theta_s
\]

\[
v
\]

\[
R
\]

(b) To calculate the Doppler shift, you need to assume the direction of the blood velocity (shown directed to the right in the above examples). If your assumption is wrong, how will that reflect in your calculated Doppler shift?
2. Cell sorter
   a. Note that \( \vec{v} \times \vec{B} \) and \( \vec{E} \) is the result of the cross product, but the direction of \( \vec{E} \) should be opposite of the potential whose polarity is determined by the direction of \( \vec{B} \).

   ![Diagram of cell sorter]

   b. From the Hall effect, \( V = \int_0^L \vec{v} \times \vec{B} \cdot d\vec{l} \).

   This can be treated like a parallel plate capacitor, \( C = \varepsilon_0 \varepsilon_r \frac{A}{d} = \frac{q}{V} \) where \( \varepsilon_0 \) is the electric constant (\( \approx 8.854 \times 10^{-12} \text{ F/m} \)), \( \varepsilon_r \) is the relative permittivity, \( A \) is the surface area of the plates, \( d \) is the distance between the plates, \( q \) is charge, and \( V \) is voltage.

   Rearranging leads to \( \frac{q}{A} = \varepsilon_0 \varepsilon_r \frac{V}{d} = \varepsilon_0 \varepsilon_r \vec{v} \cdot \vec{B} \)

   Substitute, \( \frac{q}{A} = 8.854 \times 10^{-12} \frac{F}{m} \times 80 \times \frac{100 \text{cm}^3}{s} \times \frac{m}{\pi (0.25 \text{cm})^2} \times \frac{m}{100 \text{cm}} \times 0.25 \frac{V \cdot s}{m^2} \)

   Resulting in a charge density of \( \frac{q}{A} = 9.019 \times 10^{-10} \frac{F}{m^2} = 9.019 \times 10^{-10} \frac{C}{m^2} \)

   Each cell is either -1 or +1 charge (coulomb), which means cell density is very low at 9.019 \( \times 10^{-10} \frac{\text{cells}}{m^2} \)

   Also acceptable:
   1 electron \((-1) \approx -1.602 \times 10^{-19} \text{ C} \) and 1 proton \((+1) \approx 1.602 \times 10^{-19} \text{ C} \)

   Cell density is about \( 5.629 \times 10^9 \frac{\text{cells}}{m^2} \) on both sides of the tube.

3. Doppler shifts
   a. Different orientations
      i. \( \Delta f_1 = f_O - f_S = -f_S \frac{v}{c} \cos \theta_S \)
      \( \Delta f_2 = f_R - f_O = f_O \frac{v}{c} \cos \theta_R \)
      \( \Delta f = \Delta f_1 + \Delta f_2 = -f_S \frac{v}{c} \cos \theta_S + \frac{v}{c} \cos \theta_R \left( f_S - f_S \frac{v}{c} \cos \theta_S \right) \)
      \( \Delta f = f_S \frac{v}{c} \left( -\cos \theta_S + \cos \theta_R \left( 1 - \frac{v}{c} \cos \theta_S \right) \right) \)

   Also acceptable:
Since $\frac{\nu}{c} \ll 1$, then $\Delta f = f_s \frac{\nu}{c} (-\cos \theta_s + \cos \theta_R)$

ii. $\Delta f_1 = f_o - f_s = -f_s \frac{\nu}{c} \cos \theta$

$\Delta f_2 = f_R - f_o = -f_o \frac{\nu}{c} \cos \theta$

$\Delta f = \Delta f_1 + \Delta f_2 = -\frac{\nu}{c} \cos \theta (f_s + f_o) = -\frac{\nu}{c} \cos \theta \left(f_s + f_s - f_s \frac{\nu}{c} \cos \theta \right)$

$\Delta f = -2f_s \frac{\nu}{c} \cos \theta + f_s \left(\frac{\nu}{c}\right)^2 \cos^2 \theta$

iii. Same as (i)

b. The Doppler shift can be (+) or (-) depending on if the direction of the observer is moving towards or away from each other respectively. If the assumption of the direction is wrong, the resulting calculated Doppler shift will have an opposite sign of what is expected.
4. **Design problem**: Design an automated sphygmomanometer that includes a digital readout of systolic and diastolic pressure and heart rate.

You have these components available:

(a) Cuff with arbitrary digital control over applied pressure;
(b) Sonic transducer contacting skin over artery inside the cuff, with digital readout of the acquired sound waveform;
(c) Alphanumeric display;
(d) Programmable microcontroller;
(e) Start button and sleep button;
(f) Power supply with power on/off switch.

Sketch a diagram of the system, and outline the algorithm running on the microcontroller to control the cuff, acquire the sound signal, and compute and display the systolic pressure, diastolic pressure, and heart rate.

You do not need to write actual code, but be quantitative in your description with sufficient information for a programmer to be able to complete the design. You may use timing diagrams and equations to define the sequence of control variables, the corresponding measured quantities, and the computation of the output variables.

Define important parameters in the algorithm, and give numerical values based on the physiological range that you expect.
Figure from: J Abdul Sukor et al 2012 Physiol. Meas. 33 465.
1. The lab technicians are running a routine checkup on a patient’s blood sample using some electrodes connected to an arterial gas meter. The meter measures the electrodes and calculates pH, PCO$_2$, and [O$_2$]. Unfortunately, the meters are broken. Fortunately, you come in to save the day as the biomedical engineer with a multimeter to measure the electrodes directly.

(a) You measure a voltage of −50 mV across the pH probe. The probe uses a saturated HCl concentration of 100 mM. What is the blood sample’s pH?

(b) You calibrate the PCO$_2$ electrode with a standard (pH = 7.4, PCO$_2$ = 40 mmHg). Then, you put a sample of the blood from part (a) in the PCO$_2$ electrode with 25 mM NaHCO$_3$. What is the sample’s PCO$_2$?

(c) For measuring PO$_2$, you construct a transimpedance amplifier to convert the current output from the Clark electrode to a voltage:

\[
\frac{V_{\text{out}}}{I_{\text{in}}} = \frac{R_3}{R_1 + R_2 + R_3}
\]

You set $R_1 = 1 \text{ M}\Omega$, $R_2 = 50 \text{ k}\Omega$, and $R_3 = 5 \text{ M}\Omega$. Then, you flow the blood sample through a Clark electrode at 50 mL/s which has 0.7 V applied across it. You measure a voltage of 5.5 V at $V_{\text{out}}$. What is the sample’s [O$_2$]?
1) From lecture notes (#15 pg. 3)

\[ V = 62 \text{ mV} \cdot (\text{pH}_{\text{HCl}} - \text{pH}_{\text{sample}}) \]

\[ \text{v for 0.1 mol L}^{-1} \text{ HCl in glass bulb} \]

\[ -50 \text{ mV} = 62 \text{ mV} \cdot (6 - \text{pH}) \]

\[ \text{pH} = 6.8 \]

2) From lecture notes (#15 pg. 5)

\[ \log \left( \frac{\text{PCO}_2}{1 \text{ mmol}} \right) = -7.4 + k \]

\[ \log \left( \frac{40 \text{ mmol}}{1 \text{ mmol}} \right) = -7.4 + k \Rightarrow k = 9.002 \]

\[ \log \left( \frac{\text{PCO}_2}{1 \text{ mmol}} \right) = -6.8 + 9.002 \Rightarrow \text{PCO}_2 = 160 \text{ mmol} \]

Okay it different number used - application of equations is more important.

3) Effective feedback resistance:

\[ R_{\text{eff}} \approx R_1 \frac{R_3}{R_2} \]

if \[ R_1, R_3 \gg R_2 \]

( Lecture notes #15 pg. 8) \[ R_{\text{eff}} = 100 \text{ M}\Omega \]

Transconductance amp:

\[ V = -1 \text{V} \]

\[ 5.5 \text{V} = -1 (100 \text{ M}\Omega) \]

\[ I = -55 \text{ nA} \]

( polarity irrelevant to question) \[ [\text{O}_2] = 2.8 \text{ pM} \]
2. Pulse oximeters work by passing red light (660 nm) and infrared light (805 nm) through a body part (such as a finger). The absorbance is measured using a photodiode and an amplifier. The SpO$_2$ can be calculated from the ratio of the absorbances.

(a) Why do pulse oximeters use two different wavelengths to measure SpO$_2$? Be sure to mention the isobestic wavelength’s role in computing SpO$_2$.

(b) What is the origin of the DC component of the photodiode signal? What is the origin of the AC component?

(c) Shown below is a sample of the raw pulse oximeter data:

![Graph of pulse oximeter data](image)

The SpO$_2$ can be estimated by:

$$\text{SpO}_2 = \left(110 - 25 \times \frac{\text{Abs}_{660\text{ nm}}}{\text{Abs}_{805\text{ nm}}}ight) \%$$

What is the patient’s SpO$_2$, according to the raw pulse oximeter data? *Hint:* Normalize the AC components against their corresponding DC components first. The AC component is the peak–to–peak voltage and the DC component is the average voltage.

(d) What is the patient’s heart rate, according to the raw pulse oximeter data?
2)
   a) Pulse oximetry is determined via Beer's Law:
      \[ A(\lambda_i) = W \cdot L \cdot \alpha_i \]
      The 805 nm light is the isobestic wavelength,
      meaning it has the same \( \alpha_i \) as when 660 nm
      is used. This allows us to determine absorbance
      independent of \( W \) and \( L \).
   
   b) DC \to Skin, Veins, Muscle, Fat, Steady blood flow
      Ac \to Arteries, pulsating blood flow
   
   c) Normalize:
      \[
      \begin{array}{c|c|c}
      \text{AC} & \text{660 nm} & \text{805 nm} \\
      \text{DC} & 0.2 V & 0.6 V \downarrow \\
      \end{array}
      \quad \frac{2/3 \quad \downarrow \quad 3/2}{SPO_2 = 110 - 25 \times \frac{2/3}{3/2}}
      \]
      \[ SPO_2 = 98.9\% \]
   
   d) Heart rate
      \[
      \text{1 pulse} = \frac{60 \text{ bpm}}{\text{sec}}
      \]
      by inspection
      of pulse oximeter
      graph.
3. Cardiomyocytes have a typical membrane time constant of 2 ms and starts to fibrillate at above 100 mA.

   (a) Suppose an unfortunate person gets struck with 200 kV lightning. The current enters and exits the body through skin \( R_{\text{skin}} = 100 \, k\Omega \). How much current will flow through the victim? Assume the internal body resistance is negligible compared with the skin resistance.

   (b) If 20% of the lightning strike’s current mentioned in part (a) passes through the heart, how long can the victim withstand the strike before suffering cardiac arrest?

   (c) Lightning strikes aren’t the only electrical hazard to worry about. Ground faults can make medical equipment dangerous to use without any outward signs of a problem.

   You are designing a 12–lead ECG powered by a 120 VAC supply. Because you are a competent engineer, you add resistors in series with each ECG electrode lead wire as part of the device’s overall safety design. Using the graph below, specify what resistor value you should use so that any ground fault currents are imperceptible.

   ![Graph of AC-1 to AC-4.3 showing imperceptible to muscle contraction with reversible effects to possible irreversible effects]

AC-1  imperceptible
AC-2  perceptible but no muscle reaction
AC-3  muscle contraction with reversible effects
AC-4  possible irreversible effects
   AC-4.1  up to 5% probability of heart fibrillation
   AC-4.2  5-50% probability of fibrillation
   AC-4.3  over 50% probability of fibrillation

*Graph source: International Electrotechnical Commission (IEC) standard 60479*
3) a) \[ V = IR \]
\[ 200 \text{ KV} = I (100 \text{ k\Omega} + 100 \text{ k\Omega}) \]
\[ I = 1 \text{ A} \]

b) \[ 20\% \text{ of } 1 \text{ A} = 200 \text{ mA} \]
Use rheobase equation to determine threshold.
\[ I_d = I_r \frac{1}{1-e^{-d/2}} \]
\[ Z = 2 \text{ ms} \text{ (given)} \]
\[ 200 \text{ mA} = 100 \text{ mA} \frac{1}{1-e^{-d/2(\text{ms})}} \]
Fibrillation current for any duration \( d \)

\[ d = 1.386 \text{ ms} \]

c) For imperceptible ground fault currents, look at region AC-1 on graph.
Max current for AC-1: 0.5 mA
Max possible voltage: 120 V AC
\[ V = IR \]
\[ 120 V = (0.5 \text{ mA}) R \Rightarrow R = 240 \text{ k\Omega} \]
4. **Design problem:** A group of athletes would like to know their metabolic rates during exercise. A non–invasive way to estimate this is to measure the CO₂ and O₂ concentration in their breath as they train.

Your task is to design a circuit to interface a Severinghaus electrode (measures PCO₂) and a Clark electrode (measures PO₂) to a microcontroller. This microcontroller is already loaded with software that computes the metabolic rate based on PCO₂ and PO₂. You have:

- The Severinghaus electrode. Its usable output range is $-250$ mV to $250$ mV.
- The Clark electrode. Its usable output current is $0$ μA to $100$ μA. This electrode needs $0.7$ V applied to it.
- Op–amps, and any resistors/capacitors as needed.
- A stable $\pm 5$ V power supply.
- A microcontroller with a built–in ADC. The ADC can only measure positive voltage.

The circuit needs to filter out signals above $10$ Hz from both sensors. The ADC should read $0$ V to $5$ V linearly with respect to each electrode’s usable output. You must include numerical values in your design. **BONUS:** Design your circuit in such a way that a $-5$ V supply is not required.
4) Design problem: possible solution

Biasing circuit

CO₂ probe

High impedance input

Gain = 10

Non-inverting amp

0-5V

Frequency: f_c = 10 Hz

LP filter

Biasing circuit

O₂ probe

Gain = -50 kΩ

Transimpedance amp

Frequency: f_c = 10 Hz

LP filter

Microcontroller

Display