BENG 186B Winter 2018

Quiz 1

Friday, January 26, 2018

Name (Last, First): SOLUTIONS

- This quiz is closed book and closed notes. You may use a calculator for algebra and arithmetic.
- Do not attach separate sheets. If you need more space, use the back of the pages.
- Circle or box your final answers and show your work on the pages provided.
- There are 4 problems. Points for each problem are given in [brackets]. There are 100 points total.
- You have 50 minutes to complete this quiz.
1. [10 pts] Circle the best answer (only one answer per question):

(a) [2.5 pts] Indirect physiological measurements are typically:
   i. higher bandwidth.
   ii. more accurate.
   iii. more expensive.
   iv. less invasive.

(b) [2.5 pts] The transfer function of a critically damped second-order low-pass filter has:
   i. two complex conjugate poles.
   ii. two identical real poles.
   iii. one real zero and one real pole.
   iv. one real zero and one imaginary pole.

(c) [2.5 pts] The gauge factor of a strain gauge is independent of:
   i. piezo-resistive effect.
   ii. Poisson’s ratio.
   iii. Young’s modulus.
   iv. temperature.

(d) [2.5 pts] A linear variable differential transformer is a type of inductive sensor that offers:
   i. zero offset.
   ii. greater linearity.
   iii. greater noise suppression.
   iv. all of the above.
2. [25 pts] Derive the Thévenin equivalent at node $V$ in the circuit below:

(a) [10 pts] Find the Thévenin equivalent open-circuit voltage $V_{oc}$.

The voltage-dependent current source $GV$, with voltage $V$ across it, is equivalent to a resistance $1/G$:

$$
GV \quad \overset{+}{V} \quad \equiv \quad \frac{1}{G} \quad \overset{+}{0} \quad \overset{-}{V} \quad \overset{-}{0}
$$

$$
\Rightarrow \quad V_s \quad \overset{+}{\frac{1}{G}} \quad || \quad R_2 = \frac{\frac{1}{G} \cdot R_2}{\frac{1}{G} + R_2} = \frac{R_2}{1 + R_2G}
$$

$$
V_{oc} = \frac{R_2}{1 + R_2G} \frac{V_s}{R_2 + R_1(1 + R_2G)} V_s = \frac{R_2}{R_1 + R_2 + R_1R_2G} V_s
$$
(b) [10 pts] Find the Thévenin equivalent impedance $Z_{th}$.

Kill the independent source $V_s$:

\[
\begin{align*}
\frac{R_1}{1 + \frac{R_2}{1 + R_2G}} = \frac{R_2}{1 + R_2G} \\
Z_{th} &= R_1 \parallel \frac{R_2}{1 + R_2G} \\
&= \frac{\frac{R_1R_2}{R_1 + \frac{R_2}{1 + R_2G}}}{R_1 + \frac{R_2}{1 + R_2G}} = \frac{R_1R_2}{R_1(1 + R_2G) + R_2}
\end{align*}
\]

\[
\begin{align*}
&= \frac{R_1R_2}{R_1 + R_2 + R_1R_2G}
\end{align*}
\]

(c) [5 pts] Draw the Thévenin equivalent diagram.

\[
\begin{align*}
Z_{th} &= \frac{R_1R_2}{R_1 + R_2 + R_1R_2G} \\
V_{oc} &= \frac{R_2}{R_1 + R_2 + R_1R_2G} V_s
\end{align*}
\]
3. \textbf{[35 pts]} Consider the voltage-input, current-output filter circuit below.

(a) \textbf{[10 pts]} Find the input impedance \( Z_{in}(j\omega) \).

\[ Z_{in}(j\omega) = \frac{1}{j\omega C_1} \parallel \frac{1}{j\omega C_2} \parallel R_2 \]

\[ = \frac{1}{j\omega C_1 + j\omega C_2 + \frac{1}{R_2}} \]

\[ = \frac{R_2}{1 + j\omega R_2 (C_1 + C_2)} \]
(b) [10 pts] Find the output impedance $Z_{\text{out}}(j\omega)$.

\[
\text{Kill the ideal voltage source @ input } \Rightarrow V_{\text{in}} = 0
\]

\[
Z_{\text{out}}(j\omega) = \frac{1}{j\omega C_2} \left/ \left( R_2 \right/ R_3 \right) \right.
\]

\[
= \frac{1}{j\omega C_2 + \frac{1}{R_2} + \frac{1}{R_3}}
\]

\[
= \frac{R_2 \cdot R_3}{j\omega R_2 + R_3 C_2 + R_2 + R_3}
\]

\[
= \frac{R_2 R_3}{R_2 + R_3}
\]
(c) [5 pts] Find the transfer function $H(j\omega) = \frac{I_{\text{out}}(j\omega)}{V_{\text{in}}(j\omega)}$.

\[ V_{\text{in}} \rightarrow \frac{C_2}{R_2} \rightarrow I_{\text{out}} \rightarrow V_{\text{out}} = 0 \]

\[ I_{\text{out}} = j\omega C_2 V_{\text{in}} + \frac{V_{\text{in}}}{R_2} \]

\[ H(j\omega) = j\omega C_2 + \frac{1}{R_2} \]

(d) [10 pts] Sketch the Bode plot of the transfer function $H(j\omega)$ for $C_1 = 100$ nF, $C_2 = 10$ \(\mu\)F, $R_2 = 1$ k\(\Omega\), and $R_3 = 100$ k\(\Omega\). Be sure to label the axes and indicate the units (rad/s, dB\(\Omega\)^{-1}, and degrees).

- No poles
- Single zero at $-\frac{1}{R_2 C_2} = -100 \frac{\text{rad}}{s}$
- $|H(j\omega)| \rightarrow \frac{1}{R_2} = 10^{-3} \frac{1}{\text{rad}}$ for $\omega \rightarrow 0$

$H(j\omega)$ Magnitude

$H(j\omega)$ Phase
4. **[30 pts]** Consider the stress transducer below, with constant supply voltage $V_i = 3 \, \text{V}$, two constant resistors each with resistance $R = 100 \, \text{k}\Omega$, and two strain gauges $R_{G1}$ and $R_{G2}$ with identical nominal resistance $R_{\text{nom}}$ and gauge factor $G = -100$ that are differentially activated by complementary strain due to complementary stress $\sigma$ and $-\sigma$ as shown. Both strain gauges have the same Young’s modulus $E = 100 \, \text{kPa}$.

![Diagram of the stress transducer](image)

$$R_{G1} = R_{\text{nom}} (1 - G\epsilon) = R_{\text{nom}} \left(1 - \frac{G}{E} \sigma\right)$$

$$R_{G2} = R_{\text{nom}} (1 + G\epsilon) = R_{\text{nom}} \left(1 + \frac{G}{E} \sigma\right)$$

$$\sigma = E \epsilon$$

(a) **[10 pts]** Find the output voltage $V_o$ as a function of stress $\sigma$.

$$V_o = \left(\frac{R_{G2}}{R_{G2} + R} - \frac{R_{G1}}{R_{G1} + R}\right) V_i$$

$$= \left(\frac{R_{\text{nom}} \left(1 + \frac{G}{E} \sigma\right)}{R_{\text{nom}} \left(1 + \frac{G}{E} \sigma\right) + R} - \frac{R_{\text{nom}} \left(1 - \frac{G}{E} \sigma\right)}{R_{\text{nom}} \left(1 - \frac{G}{E} \sigma\right) + R}\right) V_i$$
(b) [10 pts] Find the value of $R_{nom}$ that maximizes the sensitivity of the stress transducer, for low levels of stress $\sigma \approx 0$.

Sensitivity for the Wheatstone bridge is maximum for $R_{nom} = R = kR$

Sensitivity:

$$2 \cdot \frac{1}{4} \frac{G}{E} V_i$$

$$= \frac{1}{2} \frac{G}{E} V_i$$

$$= -\frac{1}{2} \frac{100}{100kPa} \cdot 3V = -1.5 \frac{V}{kPa}$$
(c) [10 pts] A 10-bit analog-to-digital converter (ADC) is used to digitize the voltage output $V_o$ for a digital reading of stress $\sigma$. The full-scale voltage range of the ADC is from 1 V to 2 V. Find the accuracy of the stress reading for low levels of stress $\sigma \approx 0$.

Voltage resolution: $1 \text{ LSB} = \frac{1 \text{ V}}{1.024} = 1 \text{ mV}$

Stress resolution: $1 \text{ LSB} = \frac{1 \text{ mV}}{1.5 \text{ V/MPa}} = 0.66 \text{ Pa}$

Worst-case accuracy: $\frac{1}{2} \text{ LSB} = 0.33 \text{ Pa}$