This quiz is closed book and closed notes. You may use a calculator for algebra.

Circle your final answers in the space provided; show your work only on the pages provided.

Do not attach separate sheets. If you need more space, use the back of the pages.

There are 4 problems. Points for each problem are given in brackets. There are 100 points total. You have 50 minutes to complete this quiz.
1. [35 pts] Consider the instrumentation amplifier (IA) for biopotential sensing shown below. The IA is connected to electrodes, LA, RA, and RL to measure lead I of an electrocardiogram. The body is driven to ground through electrode RL. Assume that the differential amplifier $A_{\text{AMP}}$ included as part of the IA has a differential gain of 20 [V/V] and common-mode gain of 0.1 [V/V], with infinite input impedance. The operational amplifiers $A_{\text{BUF}}$ and $A_{\text{DRL}}$ are ideal. Each electrode has a resistance of 100 kΩ. $R_1 = 1$ kΩ, $R_2 = 50$ kΩ, $R_3 = R_4 = 200$ kΩ, $R_f = 100$ MΩ, and $R_{\text{safe}} = 5$ MΩ.
a) [10 pts] Find the amplitude of the common-mode interference $V_{cm}$ for a displacement current entering the body with amplitude 1 $\mu$A.
b) [15 pts] A differential ECG signal $V_d$ of 1 mV amplitude is present at the input. Find the signal-to-noise (SNR) ratio at $V_{OUT}$. (Hint: Find the differential gain $A_d$ and the common-mode gain $A_{cm}$ of the instrumentation amplifier to formulate your answer, where $V_{OUT} = A_d V_d + A_{cm} V_{cm}$)
c) [5 pts] Now consider a 10% mismatch between $R_3$ and $R_4$ where $R_3 = 220\, \text{k}\Omega$ and $R_4 = 180\, \text{k}\Omega$. Will this cause the differential gain $A_d$ of the IA to increase, decrease, or stay the same? Explain why.

d) [5 pts] The IA uses dual supply voltages, ±5V. Suppose the DRL amplifier fails and $R_f$ becomes an open circuit. This causes the DRL to inject current into the body through the RL electrode. What is the maximum amount of current that it can inject?
2. **[25 pts]** You have a device called The Bloody Brilliant™ Tricorder (BBT) which can measure many different cardiovascular vital signs. However, its numerical readout is not working despite its bloody brilliance. Fortunately, its graphical display does work. Shown below are the blood pressure monitor readings from the BBT.

**Korotkoff Sound Level**

![Korotkoff Sound Level Graph](image)

**Arm Cuff Air Pressure**

![Arm Cuff Air Pressure Graph](image)

a) **[5 pts]** What is the systolic and diastolic blood pressure according to the above graphs?
The BBT can measure blood velocity using the Doppler principle. The BBT mixes the ultrasonic source signal with the received signal, producing a “beating” pattern shown below. (Not to be confused with heartbeats. This is a completely different phenomenon.) The number of beats per second is the frequency difference between the source and received signal.

\[
\text{Amplitude of } (f_{\text{received}} - f_{\text{source}}) \text{ signal}
\]

b) [10 pts] The source produces a 1 MHz signal. Both the source and the receiver are angled 60° from the blood vessel. The speed of sound in blood is about 1500 m/s. What is the blood velocity? 

**Hint:** the following equation may be useful:

\[
\Delta f = \frac{v}{c} (\cos \theta_{\text{source}} + \cos \theta_{\text{receiver}}) f_{\text{source}}
\]
The BBT also includes a pulse oximeter. Its readings are shown below.

**Pulse Oximeter Readings**

![Graph showing pulse oximeter readings at 660 nm and 805 nm wavelengths](image)

**Absorbance units**

**Time / seconds**

(c) [5 pts] What is the pulse *in beats per minute* according to the pulse oximeter?

(d) [5 pts] For this particular BBT, the SO₂ is calculated as:

\[
SO₂ = \left(130 - 90 \times \frac{\text{Absorbance}_{660\ nm}}{\text{Absorbance}_{805\ nm}}\right) \%
\]

What is the *minimum* SO₂?
3. **[20 pts]** Circle the best answer (only one answer per question):

The electrical equivalent of the pressure-fluid-diaphragm (without bubbles) is shown in the figure below. The equations for the resistive and inertial components of the system are shown next to the circuit.

\[ R = \frac{8\eta l}{\pi r^4} \quad L = \frac{\rho l}{\pi r^2} \]

a) **[4 pts]** If the viscosity of the fluid inside the catheter is almost negligible (yet, with a non-zero density), this system tends to become:

i. oscillatory  
i. under-damped  
i. over-damped  
i. critically-damped  
v. None of the above

b) **[4 pts]** When the inertial characteristics of the fluid inside the catheter are negligible, the transfer function of the system approaches a(an):

i. high-pass filter  
i. band-pass filter  
i. low-pass filter  
i. oscillator  
v. None of the above

c) **[4 pts]** The ratio of the change in volume by the change in pressure is:

i. resistance  
i. inertance  
i. compliance  
i. flow  
v. None of the above
d) [8 pts] Indicate whether each of the following statements is true or false:

[ TRUE / FALSE ] In the three electrode configuration for ECG recordings, lead I receives this nomenclature since it is where the cardiac vector $\vec{M}$ is most observed.

[ TRUE / FALSE ] The ventricular depolarization is represented by the QRS complex.

[ TRUE / FALSE ] A signal-to-noise ratio of 40dB indicates a signal amplitude $10^4$ times that of the noise.

[ TRUE / FALSE ] In an instrumentation amplifier, a lower common-mode gain results in a higher common-mode rejection ratio.

[ TRUE / FALSE ] The active grounding of the body by common-mode feedback is also known as “driven right leg”.

[ TRUE / FALSE ] The damping ratio of a fluid-filled catheter is directly proportional to the resistance of fluid flow inside the catheter.

[ TRUE / FALSE ] The partial pressure of $O_2$ in the blood is equivalent to the saturation of $O_2$ in hemoglobin.

[ TRUE / FALSE ] Between the systolic and diastolic blood pressures, the Korotkoff sounds are present.
4. **[20 pts]** The following system is used to measure the pH of a sample solution. The reference electrode (A) presents a half-cell potential of $E_A = 0.25 \text{ mV}$ and the $H^+$-sensitive electrode (B) presents a half-cell potential of $E_B = 0.75 \text{ mV}$.

$$E_{Nernst} = 60 \text{ mV} \times \log_{10} \frac{[\text{Ion}^+]_o}{[\text{Ion}^+]_i}$$

a) **[5 pts]** Given the concentration of the saturated HCl solution in the glass bulb is 10 mmol/liter, what is the pH of this solution?

b) **[5 pts]** Calculate the Nernst potential ($E_{Nernst}$) for a sample solution with $[H^+] = 10^{-4} \text{ mol/liter}$. 
c) [5 pts] What is the pH of the sample solution when $E_{Nernst} = 60$ mV?

d) [5 pts] Obtain the voltage $V_{el}$ measured across the electrodes in the case where the pH of the sample solution is the same as that inside the bulb.