BENG 186B Winter 2013
Quiz 3

March 8, 2013

NAME (Last, First):

• This quiz is closed book and closed note. You may use a calculator for algebra.

• Write your final answers in the space provided; show your work only on the pages provided.

• Do not attach separate sheets. If you need more space, use the back of the pages.

• Points for each problem are given in [brackets], 100 points total. The quiz is 50 minutes long.
1. [15 pts] Using Einthoven’s Triangle, write the lead voltages I, II, III, aVR, aVL, and aVF in terms of the electrode voltages RA, LA, and LL.

\[
\begin{align*}
I &= \frac{1}{15}RA - \frac{1}{40}LA - \frac{1}{50}LL \\
II &= \frac{2}{5}RA - \frac{1}{10}LA + \frac{1}{25}LL \\
III &= \frac{3}{20}RA - \frac{1}{20}LA + \frac{2}{50}LL \\
aVR &= \frac{3}{25}RA - \frac{1}{25}LA + \frac{1}{50}LL \\
aVL &= \frac{3}{25}RA + \frac{1}{25}LA - \frac{1}{50}LL \\
aVF &= \frac{3}{25}RA + \frac{1}{25}LA + \frac{1}{50}LL
\end{align*}
\]
2. [40 pts] Consider a single-lead ECG instrumentation amplifier with driven right leg (DRL), and with three electrodes RA, LA and RL connected to the body as shown below. The electrode-skin interface resistances are $R_{RA} = 50 \, \text{k}\Omega$, $R_{LA} = 150 \, \text{k}\Omega$ and $R_{RL} = 100 \, \text{k}\Omega$. The opamp input impedance is $R_{in} = 1 \, \text{G}\Omega$ to ground. The instrumentation amplifier resistance values are $R_2 = R_3 = 1 \, \text{k}\Omega$ and $R_1 = R_4 = 100 \, \text{k}\Omega$, and the DRL resistance values are $R_a = 100 \, \text{k}\Omega$, $R_f = 10 \, \text{M}\Omega$, and $R_o = 1 \, \text{M}\Omega$, all exactly (with zero tolerance).

![ECG Instrumentation Amplifier Circuit Diagram]

a. [5 pts] What ECG lead does $V_{out}$ represent? What is the polarity of the lead in $V_{out}$?

Answer:

ECG Lead = _____  

Polarity = _____ (+/-)
b. [15 pts] Find the values for differential gain $A_d$ and common-mode rejection ratio CMRR of the instrumentation amplifier connected to the RA and LA electrodes on the body.

Answer:

$$A_d = \text{__________} \quad \text{CMRR} = \text{__________}$$
c. [10 pts] Find the driven right leg (DRL) effective electrode resistance, defined as $R_{\text{RL eff}} = \frac{v_{cm}}{i_d}$.

Answer:

$$R_{\text{RL eff}} = \phantom{} \text{k}\Omega$$
d. [10 pts] Assume a displacement current of peak amplitude $i_d = 10 \, \mu A$ from 60 Hz line noise enters the body, and an ECG differential signal of peak amplitude $v_d = 1 \, mV$ is generated by the heart. Find the amplitude of the resulting 60 Hz common-mode component of the voltage output $V_{out \, cm}$, and the corresponding signal-to-noise ratio $\text{SNR}_{out}$ at the output.

Answer:

$$V_{out \, cm} = \text{___________} \quad \quad \quad \text{SNR}_{out} = \text{___________}$$
3. [20 pts] An ENT doctor uses a laser Doppler vibrometer to measure the health of the eardrum. The 800 nm wavelength laser beam reflects off the eardrum and returns to a collector located at the same place as it was emitted. The doctor produces single-pitch sound at 20 kHz that causes the eardrum to vibrate sinusoidally with peak amplitude of 100 µm. Find the peak Doppler frequency shift recorded in the returning laser light. \textit{HINT}: light wave frequency is given by \( f = \frac{c}{\lambda} \) where \( c = 3 \times 10^8 \text{ m/s} \) is the speed of light, and \( \lambda \) is the wavelength.

Doppler frequency shift:
\[
\Delta f = f_R - f_s \approx \frac{v}{c} \left( \cos \theta_s + \cos \theta_R \right) f_s
\]

Answer: \( \Delta f_{\text{peak}} = \boxed{\text{MHZ}} \)
4. [25 pts] A Severinghaus electrode for $PCO_2$ measurement is shown below. The inside compartment of the glass electrode contains a solution of 0.01 mol/L HCl, and the buffer contains a 0.1 mol/L solution of NaHCO$_3$. The two Ag/AgCl electrodes are identical with half potential $E_{Ag/AgCl} = 0.223$ mV. The membrane between the sample and buffer is permeable to CO$_2$ only, equalizing the concentration on both sides.

![Severinghaus electrode diagram]

The Nernst potential for ion type $A^n$ of valance $n$ at room temperature is given by:

$$E_{Nernst} = \frac{60mV}{n} \cdot \log_{10} \left( \frac{[A^n]_o}{[A^n]_i} \right)$$

and the relationship between $PCO_2$ and $[H^+]$ and $[HCO_3^-]$ in the buffer at room temperature is given by:

$$PCO_2 = \frac{100 \text{ mmHg}}{(\text{mol/L})^2} \cdot [H^+] \cdot [HCO_3^-]$$

a. [10 pts] Write $V_{out}$ as a function of the pH of the buffer solution.

Answer: $V_{out} =$ ________________________________
b. [10 pts] Write \( V_{out} \) as a function of the \( PCO_2 \) of the sample solution.

Answer: \( V_{out} = \) ________________________________
c. [5 pts] For a given $PCO_2$, does the output $V_{out}$ depend on the flow rate of the blood sample in and out of the instrument? Does it depend on the temperature of the instrument? Explain.

Answer: $V_{out}$ depends on sample flow rate: __________

$V_{out}$ depends on temperature: __________