NAME (Last, First):

• This quiz is closed book and closed note. You may use a calculator for algebra.

• Circle your final answers in the space provided; show your work only on the pages provided.

• Do not attach separate sheets. If you need more space, use the back of the pages.

• Points for each problem are given in [brackets], 100 points total. The quiz is 50 minutes long.
1. [35 pts] Consider a bioinstrumentation amplifier with a differential gain $A_d = 100$, a practically infinite CMRR of 120 dB, and input impedance $R_{in} = 1 \, \text{M}\Omega$. We want to connect the amplifier to measure an ECG signal using three electrodes RA, LA and LR in contact with the body. The electrode contact impedances are $R_{RA} = 110 \, \text{k}\Omega$, $R_{LA} = 90 \, \text{k}\Omega$ and $R_{RL} = 100 \, \text{k}\Omega$.

a. [15 pts] Find the differential gain $A_{d1}$, common-mode gain $A_{c1}$ and CMRR$_1$ of the overall system with the bioinstrumentation amplifier connected to the RA and LA electrodes on the body.

Answer:  

$A_{d1} =$ \underline{\hspace{1cm}} \\

$A_{c1} =$ \underline{\hspace{1cm}} \\

CMRR$_1 =$ \underline{\hspace{1cm}}
b. [10 pts] Assume a displacement current of amplitude \( i_d = 1 \, \mu\text{A} \) from 60 Hz line noise entering the body. The RL electrode is connected to ground. Find the amplitude of the resulting 60 Hz common-mode voltage \( v_{\text{cm}} \) in the body and the corresponding common-mode voltage \( v_{\text{cm,\text{out}}} \) at the amplifier output.

Answer: \( v_{\text{cm}} = \_\_\_\_\_\_\_\_\_\_ \)
\( v_{\text{cm,\text{out}}} = \_\_\_\_\_\_\_\_\_\_ \)
c. [10 pts] Now the RL electrode is disconnected from ground, and a second amplifier is inserted to reduce the common-mode voltage using the driven right leg technique. The inputs of the second amplifier are connected to the LA and RA electrodes, and its output drives the RL electrode. Find the required common-mode gain $A_{c2}$ of this amplifier such that $v_{cm}$ in the body is reduced to 100 $\mu$V in amplitude.

Answer: $A_{c2} =$ _______________
2. [20 pts] Estimate, and sketch on the Einthoven’s triangles below, the direction and relative magnitude of the cardiac vectors $M$ at the ECG P, R and T waves. Show your reasoning. Hint: Try to look for leads that are approximately parallel and orthogonal to the cardiac vector.

\[
\begin{align*}
\text{I} & \quad \text{aVR} \\
\text{II} & \quad \text{aVL} \\
\text{III} & \quad \text{aVF}
\end{align*}
\]
3. **[20 pts]** Circle the best answer (circle only one letter for each question).
   
a. **One advantage of using intravascular pressure monitoring instruments as opposed to external methods is:**
   - i. Cheaper costs, reusability
   - ii. Less invasive
   - iii. Less affected by blood viscosity
   - iv. Better frequency response
   - v. Easy to sterilize

b. **The following is NOT modeled as a capacitance in a hydraulic system’s electrical analog:**
   - i. The presence of bubbles in the catheter
   - ii. The compressibility of the fluid
   - iii. The viscosity of the fluid
   - iv. The elasticity of the catheter
   - v. The compliance of the diaphragm

c. **Tonometry:**
   - i. Uses sonic tones of different frequencies to measure flow velocity
   - ii. Images by reconstructing from sections generated by a penetrating wave
   - iii. Is a direct way to measure pressure
   - iv. Measures deformation at the interface
   - v. Requires an injection of a contrast agent or thermal solution

d. **Electromagnetic flow sensors work by the following physical principle:**
   - i. Electrical induction
   - ii. Electrical capacitance
   - iii. Coriolis force
   - iv. Fick’s law
   - v. No-slip boundary condition

e. **The Doppler frequency shift in ultrasonic velocity measurement is NOT directly proportional to:**
   - i. Angle cosines between sound velocity and flow velocity
   - ii. Speed of sound
   - iii. Speed of the fluid
   - iv. Emittted frequency
   - v. Any of the above
4. [25 pts] A membrane separates a container into two compartments as shown below. The inside compartment contains a solution of 0.1 mol/L HCl, and a sample solution is contained in the outside compartment. Two identical Ag/AgCl electrodes are immersed, one in each compartment. The membrane between the compartments is permeable to H\(^+\) ion only.

![Diagram of membrane system]

The Nernst potential equation for ion A\(^n\) where n is valance of the ion:

\[
E = \frac{60mV}{n} \cdot \log_{10} \left( \frac{[A^n]_o}{[A^n]_i} \right)
\]

a. [10 pts] Derive an expression for V as a function of the pH of the sample solution.

Answer: \(V = \) _____________________
b. [5 pts] Find the value of V for a sample of 0.01 mmol/L KOH in pure water.

Answer: $V = \underline{\quad\quad\quad\quad}$
c. [10 pts] Now find V after adding 0.01 mmol/L H₂SO₄ to the KOH solution in b.

Answer: V = ______________