BENG 186B Winter 2013
Quiz 2

February 15, 2013

NAME (Last, First):

SOLUTIONS

• This quiz is closed book and closed notes. You may use a calculator for algebra.

• Circle your final answers in the space provided; show your work only on the pages provided.

• Do not attach separate sheets. If you need more space, use the back of the pages.

• Points for each problem are given in [brackets], 100 points total. The quiz is 50 minutes long.
1. [25 pts] Consider the following active circuit with voltage input $V_{in}$ and voltage output $V_{out}$. You may assume that the op-amp is ideal.

![Circuit Diagram]

a. Derive the transfer function $H(j\omega) = \frac{V_{out}(j\omega)}{V_{in}(j\omega)}$. [10 pts]

\[
\frac{V_{out}(j\omega)}{V_{in}(j\omega)} = \frac{R_2 + (R_3 || \frac{1}{j\omega C})}{R_2} = 1 + \frac{R_3 \cdot \frac{1}{j\omega C}}{R_2 \left( R_3 + \frac{1}{j\omega C} \right)}
\]

\[
= 1 + \frac{R_3}{R_2} \cdot \frac{1}{1 + j\omega R_3 C}
\]
b. Identify the pole(s) and/or the zero(s) of the transfer function \( H(j\omega) \), and sketch the Bode plot (log amplitude and phase of \( H(j\omega) \) as a function of log radial frequency \( \omega \)). [8 pts]

\[
H(j\omega) = 1 + \frac{R_3}{R_2} \cdot \frac{1}{1 + j\omega R_3 C} = \frac{(1 + \frac{R_3}{R_2}) + j\omega R_3 C}{1 + j\omega R_3 C}
\]

\[
\Rightarrow \text{pole at } j\omega = -\frac{1}{R_3 C}
\]

\[
\text{zero at } j\omega = -\frac{1 + \frac{R_3}{R_2}}{R_3 C} = -\left(\frac{1}{R_2} + \frac{1}{R_3}\right) \frac{1}{C}
\]
c. Derive the input impedance $Z_{in}(j\omega)$. [4 pts]

$$Z_{in} = \infty \quad \text{since the opamp does not draw input current (ideal opamp)}.$$


d. Derive the output impedance $Z_{out}(j\omega)$. [3 pts]

$$Z_{out} = 0 \quad \text{since $V_{out}$ is maintained by the ideal opamp regardless of output current (as long as it does not saturate).}$$
2. [25 pts] The following circuit generates a voltage output $V_{out}$ as shown below. The components OP27, LM311, 555 are assumed ideal, each with positive supply at +5 V and negative supply at GND (0 V). $R_1 = 2 \, \text{M}\Omega$, $R_2 = 1 \, \text{M}\Omega$, and $C = 100 \, \text{nF}$. The equations for the 555 are: $T_{high} = 0.7 \, (R_1 + R_2) \, C$ and $T_{low} = 0.7 \, R_2 \, C$.

![Circuit Diagram]

$V_0 = 0 - 1\, \text{V} \, \text{square wave} \quad (< 2\, \text{V} \, \text{always})$

$V_{out} = 0\, \text{V} \quad \text{always}$

$V_2 = +2\, \text{V} \quad \text{(constant!)}$

Sketch the waveforms for the voltages $V_1, V_2$ and $V_{out}$ on the diagrams on the next page. Show your work below.

$V_0$ (555 out) is a square wave between 0V and +5V, with $T_{high} = 0.7 \, (R_1 + R_2) \, C = 210 \, \text{ms}$, and $T_{low} = 0.7 \, R_2 \, C = 70 \, \text{ms}$.

$V_1 = \frac{10 \, R_2}{10 \, R_2 + 40 \, R_1}$

$V_0 = \frac{1}{5} \, V_0$ : the same square wave, but between 0V and +1V.
$V_2$ is $-1V$ amplified with an inverting gain of $\frac{-20k\Omega}{10k\Omega} = -2$, or $V_2 = +2V$. The 10pF capacitor is just an open circuit for DC voltages.

$V_{out}$ remains at 0V since $V_1 < V_2$ always.
3. [20 pts] Circle the best answer (circle only one choice for each question a. and b., and circle TRUE or FALSE for each part of question c. and d.).

a. [4 pts] In an active circuit using an ideal opamp, the voltage at the non-inverting input can be considered virtually identical to the voltage at the inverting input:
   i. When there is no feedback
   ii. When there is positive feedback
   iii. When there is negative feedback
   iv. When the output is saturated
   v. Through hysteresis

b. [4 pts] The following digital circuit comprised of only NAND gates is functionally equivalent to the following gate:

   \[
   Q = \overline{A + B} : \text{NOR}
   \]

   i. AND
   ii. OR
   iii. NOR
   iv. XOR
   v. XNOR

   \[
   \overline{A \cdot B} = A + B
   \]

c. [6 pts] Indicate whether each of the following statements regarding electrophysiology are true or false:

   [ TRUE ]/ [ FALSE ] Atrial depolarization occurs before ventricular depolarization in the ECG cycle.
   [ TRUE ]/[ FALSE ] The AV node triggers the onset of ECG cycle.
   [ TRUE ]/ [ FALSE ] Ventricular repolarization produces the highest voltage amplitude in a typical ECG.

d. [6 pts] Indicate whether each of the following statements regarding half-cell potentials are true or false:

   [ TRUE ]/ [ FALSE ] The Ag/AgCl electrode is polarizable.
   [ TRUE ]/[ FALSE ] The half-cell potential is independent of ionic concentrations in the solution.
   [ TRUE ]/ [ FALSE ] In a passive electrical circuit connected to two electrodes immersed in ionic solution, current flows from the electrode with highest half-cell potential to that with lowest.
4. [30 pts] Consider the circuit model for the electrode-skin interface below. The Ag/AgCl electrode has a half cell potential of +223 mV, and the ion concentrations in the gel and epidermis are given in the following table:

<table>
<thead>
<tr>
<th></th>
<th>Gel (outside skin)</th>
<th>Epidermis (inside skin)</th>
</tr>
</thead>
<tbody>
<tr>
<td>[Na⁺]</td>
<td>10 mmol/L</td>
<td>10 mmol/L</td>
</tr>
<tr>
<td>[K⁺]</td>
<td>100 mmol/L</td>
<td>1 mmol/L</td>
</tr>
<tr>
<td>[Cl⁻]</td>
<td>110 mmol/L</td>
<td>11 mmol/L</td>
</tr>
</tbody>
</table>

The Goldman-Hodgkin-Katz (GHK) equation:

\[
E = (60mV) \cdot \log_{10} \frac{\frac{P_K [K^+]_o + P_{Na} [Na^+]_o + P_{Cl} [Cl^-]_i}{P_K [K^+]_i + P_{Na} [Na^+]_i + P_{Cl} [Cl^-]_o}}
\]
a. Find the Nernst potentials for $Na^+$, $K^+$ and $Cl^-$ across the gel-epidermis interface. [4 pts]

\[ E_{Na} = (60 \text{ mV}) \log_{10} \frac{[Na^+]_o}{[Na^+]_i} = 60 \text{ mV} \cdot \log_{10} \frac{10}{10} = 0 \]

\[ E_K = (60 \text{ mV}) \log_{10} \frac{[K^+]_o}{[K^+]_i} = 60 \text{ mV} \cdot \log_{10} \frac{100}{1} = 120 \text{ mV} \]

\[ E_{Cl} = (-60 \text{ mV}) \log_{10} \frac{[Cl^-]_o}{[Cl^-]_i} = -60 \text{ mV} \cdot \log_{10} \frac{11}{11} = -60 \text{ mV} \]

b. Find the equilibrium potential $E_{se}$ across the gel-epidermis interface, assuming the boundary separating the epidermis from the gel is equally permeable to all ion types. [4 pts]

\[ E_{se} = -E_{GHK} = -(60 \text{ mV}) \log \frac{[K^+]_o + [Na^+]_o + [Cl^-]_i}{[K^+]_i + [Na^+]_i + [Cl^-]_o} \]

"$i$" is epidermis

"$o$" is gel

\[ = -60 \text{ mV} \cdot \log_{10} \frac{121}{121} = 0 \]

c. Derive the voltage between the electrode and the subcutaneous body at rest. [4 pts]

\[ E_{he} + E_{se} = +223 \text{ mV} + 0 \]

\[ = +223 \text{ mV} \]
d. Find the general expression for the impedance $Z(j\omega)$ of the electrode-skin interface, based on the AC equivalent circuit. What is the order of the system? Identify all poles and zeros. [10 pts]

\[ Z(j\omega) = \frac{R_d}{1 + j\omega R_d C_d} + \frac{R_c}{1 + j\omega R_c C_e} + R_s + R_m \]

SECOND-ORDER SYSTEM

Two poles: \( j\omega = -\frac{1}{R_d C_d} \) and \( j\omega = -\frac{1}{R_c C_e} \)

Two zeros:

\[ Z(j\omega) = \frac{R_d (1 + j\omega R_c C_e) + R_c (1 + j\omega R_d C_d) + (R_s + R_m) (1 + j\omega R_d C_d) (1 + j\omega R_c C_e)}{(1 + j\omega R_d C_d) (1 + j\omega R_c C_e)} \]

\[
\begin{align*}
\left[ \frac{R_d + R_c + R_s + R_m}{c} + j\omega \left( \frac{R_d R_c (R_s + R_m)}{c} + \frac{R_d R_c (R_s + R_m)}{c} \right) \right] \\
- \omega^2 \left( \frac{R_s + R_m}{R_c C_e} \right)
\end{align*}
\]

\[ Z(j\omega) = \frac{1}{(1 + j\omega R_d C_d) (1 + j\omega R_c C_e)} \]

\[ j\omega = -\frac{\beta \pm \sqrt{\beta^2 - 4\alpha c}}{2\alpha} \quad (\alpha(j\omega)^2 + \beta(j\omega) + c = 0) \]
e. Sketch the Bode plot of the impedance \( Z(j\omega) \) (log amplitude and phase as a function of log radial frequency \( \omega \)) for \( R_s = R_u = 1 \, \text{k}\Omega \), \( R_d = 10 \, \text{k}\Omega \), \( R_e = 1 \, \text{M}\Omega \), \( C_d = 100 \, \text{pF} \), and \( C_e = 1 \, \text{nF} \). Indicate units and key values on your axes. [8 pts]

\[
Z(0) = R_d + R_e + R_s + R_m = 1.012 \, \text{M}\Omega
\]

\[
Z(\infty) = R_s + R_m = 2 \, \text{k}\Omega
\]

poles: \( j \omega = -\frac{1}{R_d C_d} = 10^6 \, \text{rad/s} \), and \( j \omega = -\frac{1}{R_e C_e} = 10^3 \, \text{rad/s} \)

\[
|Z(j\omega)| (\Omega) = \frac{R_e + R_d + R_s + R_m}{1 + j \omega R_e C_e}
\]

\[
\angle Z(j\omega) (^{\circ}) = \frac{R_d + R_s + R_m}{1 + j \omega R_d C_d}
\]

poles:

\[
j \omega = -\frac{1}{R_e C_e}
\]

\[
j \omega = -\frac{1}{R_d C_d}
\]

zeros:

\[
j \omega = -\left(\frac{1}{\frac{R_e}{1 + j \omega R_e C_e}} + \frac{1}{R_d + R_s + R_m}\right)
\]

\[
j \omega = -\left(\frac{R_d}{1 + j \omega R_d C_d} + \frac{1}{R_s + R_m}\right)
\]