BENG 186B Winter 2013

Quiz 1

January 25, 2013

NAME (Last, First) SOLUTIONS

• This quiz is closed book, closed notes, you may use a calculator for algebra.

• Circle your final answers and show your work on the pages provided.

• Do not attach separate sheets. If you need more space, use the back of the pages.

• Points for each problem are given in [brackets], 100 points total. The quiz is 50 minutes long.
1. [25 pts] Find the Thévenin equivalent at $V_o$ for the following circuit:

The voltage on this node
is $V_S$ regardless of $R_1$.

The current $I_b$ flows into the $V_0$ node regardless of the $+5V$ and $R_4$.

$V_{TH} = \frac{V_S}{R_2} + \frac{I_b}{R_2} = \frac{1}{R_2} \frac{V_S}{R_3} + \frac{R_2 R_3 I_b}{R_2 + R_3}$
Alternative solution (among many others!):

Linear superposition:

- $I_b = 0$:
  \[ V_s \quad \frac{R_2}{R_2 + R_3} \quad V_{TH} = \frac{R_3}{R_2 + R_3} \quad V_s \]

- $V_s \neq 0$

- $I_b \neq 0$:
  \[ \frac{R_2}{R_2 + R_3} \quad \frac{R_2}{R_2 + R_3} \quad V_{TH} = \left( \frac{R_2}{R_2 + R_3} \right) \cdot I_b \]

\[
\xRightarrow{=} \quad V_{TH} = \frac{R_3}{R_2 + R_3} \quad V_s + \frac{R_2 R_3}{R_2 + R_3} \quad I_b
\]

$Z_{TH}$: Kill the source:

\[
\xRightarrow{=} \quad V_s \to 0V \quad \text{(short)}
\]

\[
I_b \to 0A \quad \text{(open)}
\]

\[
\xRightarrow{=} \quad Z_{TH} = \frac{R_2}{R_2 + R_3} + \frac{R_2}{R_2 + R_3} = \frac{R_2 R_3}{R_2 + R_3}
\]

\[
\xRightarrow{=} \quad \frac{Z_{TH}}{R_2 + R_3} = \frac{R_2 R_3}{R_2 + R_3}
\]

\[
\xRightarrow{=} \quad V_{TH} = \frac{R_3}{R_2 + R_3} \quad (V_s + R_2 I_b)
\]
2. [35 pts] For the following current-in, voltage-out filter circuit:

![Circuit Diagram]

a. Find the input impedance \( Z_{in} \).

\[ V_{in} = V_{out} ! \]

\[ \Rightarrow Z_{in} = R \parallel \frac{1}{j\omega C} = \frac{R \cdot \frac{1}{j\omega C}}{R + \frac{1}{j\omega C}} = \frac{R}{1 + j\omega RC} \]

\[ = \frac{R}{1 + j\omega Z} \quad \text{where} \quad Z = RC \]
b. Find the output impedance $Z_{out}$.

\[
\text{Ideal current source } I_{in} \text{ at the input has } \infty \text{ impedance. (open circuit to "kill" the source)}
\]

\[
V_{out} \quad \Rightarrow \quad Z_{out} = Z_{in}!
\]

\[
Z_{out} = \frac{R}{1 + j\omega Z}, \quad Z = RC
\]

c. Find the transfer function $H(j\omega) = \frac{V_{out}(j\omega)}{I_{in}(j\omega)}$.

\[
V_{in} = V_{out} \Rightarrow H(j\omega) = \frac{V_{out}}{I_{in}} = Z_{in} = Z_{out}!
\]

\[
H(j\omega) = \frac{R}{1 + j\omega Z}, \quad Z = RC
\]
d. A variable capacitance $C$, with adjustable transversal distance $x$ between conductor plates illustrated below, is used to tune the cut-off frequency of the above filter circuit, while the resistance $R$ is kept at 100 kΩ. At a distance $x = 10 \, \mu m$ the capacitance $C$ is measured to be $10 \, nF$. Find the radial cut-off frequency of the circuit for $x = 1 \, \mu m$, and for $x = 100 \, \mu m$.

\[
C = \varepsilon_0 \varepsilon_r \frac{A}{x} = \varepsilon_0 \varepsilon_r \frac{a \cdot b}{x} : \text{inversely proportional to } x
\]

\[
\begin{align*}
\text{• } x = 10 \, \mu m & : \quad C = 10 \, nF \\
\text{• } x = 1 \, \mu m & : \quad C = 100 \, nF \\
\Rightarrow \quad \frac{1}{RC} &= \frac{1}{100 \, k\Omega \cdot 100 \, nF} = 100 \, rad/s \\
\text{• } x = 100 \, \mu m & : \quad C = 1 \, nF \\
\Rightarrow \quad \frac{1}{RC} &= \frac{1}{100 \, k\Omega \cdot 1 \, nF} = 10,000 \, rad/s
\end{align*}
\]
e. Sketch the Bode plots, with magnitude and phase, of the transfer function $H(j\omega)$ for $R = 100$ kΩ and $x = 100$ μm. Be sure to label all axes with units.

![Bode Plots](image)
3. [20 pts] For the circuit below, calculate the power efficiency \( \eta \) at steady state. The power efficiency is defined as the ratio of power delivered to the load \( R_{\text{load}} \), over power delivered by the source \( V_s \).

\[ V_s \text{ is a DC source } \implies \text{ at steady state:} \]

- Short circuit: \( \frac{1}{R_C} \)
- Open circuit: \( 0 \circ 0 \circ \)

\[ I = \frac{V_s}{R_L + R_{\text{load}}} \]

\[ \eta = \frac{P_{\text{load}}}{P_{\text{source}}} = \frac{P_{\text{load}}}{P_{\text{load}} + P_{R_L}} = \frac{R_{\text{load}} I^2}{R_{\text{load}} I^2 + R_L I^2} = \frac{R_{\text{load}}}{R_{\text{load}} + R_L} \]
4. [20 pts] Consider the temperature transducer below, with a constant voltage supply $V_i = 1 \text{ V}$, two constant resistors each with resistance $R$, and two thermistors with temperature dependence $R_{Th1} = R \left(1 + k_1 T \right)$ and $R_{Th2} = R \left(1 + k_2 T \right)$ respectively, where $T$ is the ambient temperature. Find values of the temperature coefficients $k_1$ and $k_2$ for which the transducer is linear with constant sensitivity $dV_o / dT = 0.002 \text{ V/°C}$.

\[
\begin{align*}
V_o &= \left( \frac{1 + \frac{k_1 T}{2 + (k_1 + k_2)T}}{2} \right) V_i \quad \text{linear only if } k_1 + k_2 = 0 \\
\Rightarrow V_o &= \left( \frac{1}{2} - \frac{1}{2} \right) V_i = \frac{k_1 V_i}{2} T \\
\text{Sensitivity} &= \frac{dV_o}{dT} = \frac{k_1 V_i}{2} = 0.002 \text{ V/°C} \\
\Rightarrow \begin{cases} k_1 = \frac{V_i}{2} \times 0.002 \text{ V/°C} = 0.004 \text{ °C}^{-1} \\ k_2 = -k_1 = -0.004 \text{ °C}^{-1} \end{cases}
\end{align*}
\]