This final is closed book and closed notes. You may use a calculator for algebra and arithmetic.

This final has 23 pages, including this cover sheet. Do not attach separate sheets. If you need more space, use the back of the pages.

Circle or box your final answers and show your work on the pages provided.

There are 10 problems. Points for each problem are given in [brackets]. There are 100 points total.

You have 3 hours to complete this final.

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Total /100
You may find the following equations useful:

\[ \omega_n = \frac{1}{\sqrt{LC}} \quad \zeta = \frac{1}{2}RC\omega_n \]

\[ R = R_G (1 + G\epsilon) \quad \sigma = E\epsilon \]

\[ V_o = A_d V_d + A_c V_{cm} \]

\[ V_d = V_b - V_a \]

\[ V_{cm} = (V_b + V_a)/2 \]

\[ V = \mathbf{M} \cdot \mathbf{r} = \cos \theta |\mathbf{M}| |\mathbf{r}| \]

\[ e = \int_0^L \mathbf{v} \times \mathbf{B} \, d\ell \]

\[ \Delta f = \frac{v}{c} (\cos \theta_r + \cos \theta_s) f_s \]

\[ V = E_{glass} - E_{ref} + E_{Nernst} \]

\[ E_{Nernst} = \frac{RT}{nF} \ln(10) \log_{10} \left( \frac{[A^n]_{\text{out}}}{[A^n]_{\text{in}}} \right) \text{ for some ion } A^n \text{ with valence } n \]

\[ V_m = \frac{RT}{F} \ln(10) \log_{10} \frac{P_{Na}[Na^+]_o + P_{K}[K^+]_o + P_{Cl}[Cl^-]_i}{P_{Na}[Na^+]_i + P_{K}[K^+]_i + P_{Cl}[Cl^-]_o} \]

\[ \frac{RT}{F} \ln(10) = 62 \text{ mV at room temperature} \]

\[ I = 4F[O_2] \phi \quad F = 96485 \text{ C/mol} \]

\[ \log_{10} \text{PCO}_2 = -\text{pH} + \text{constant} \]

\[ O_2 + 2H_2O + 4e^- \rightleftharpoons 4OH^- \]

\[ 4Ag + 4Cl^- \rightleftharpoons 4AgCl + 4e^- \]
The following diagrams may come handy as well:

\[
T_{hi} = 0.7(R_A + R_B)C \quad T_{lo} = 0.7R_B C \quad T = 1.1RC
\]
1. [16 pts] Circle the best answer (only one answer per question):

(a) [1 pt] An analog-to-digital converter (ADC) with 1.023 V range and 1 mV LSB step has: What is the minimum number of bits the ADC needs to output?
   i. 9-bit accuracy
   ii. 10-bit resolution
   iii. 10-bit precision
   iv. 11-bit sensitivity
   v. none of the above

(b) [1 pt] The circuit shown below implements what function?

   i. Oscillator
   ii. Astable multivibrator
   iii. Monostable single-shot
   iv. Hysteretic comparator
   v. None of the above

(c) [1 pt] A cell with equal permeabilities to Na\(^+\), K\(^+\) and Cl\(^-\) and with net charge neutrality both inside and outside has a rest potential of:
   i. +80 mV
   ii. +10 mV
   iii. 0 mV
   iv. −20 mV
   v. None of the above

(d) [1 pt] Touch sensation in the peripheral nervous system is electrically measured by the:
   i. ENG
   ii. EMG
   iii. ERG
   iv. EOG
   v. None of the above
(e) [1 pt] The fraction of oxygenated hemoglobin in the blood is measured by the:
   i. PO₂
   ii. PCO₂
   iii. SCO₂
   iv. SO₂
   v. None of the above.

(f) [1 pt] The Clark electrode:
   i. Measures saturation of oxygen in the blood.
   ii. Measures the electrocardiogram non-invasively.
   iii. Causes outward current from the postsynaptic neuron generating a local current source.
   iv. Produces hydrogen peroxide in amperometric sensing of oxygen flow.
   v. None of the above.

(g) [1 pt] Pulse oximetry:
   i. Is an optical technique for determining PO₂ in blood.
   ii. Is a process of regulating oxidation in a polarized electrode by periodically reverting polarity.
   iii. Measures fluorescence by pulsing IR light.
   iv. Non-invasively measures the cardiac pulse in the blood.
   v. All of the above.

(h) [1 pt] Macroshock:
   i. Is a critical electrocution condition in which massive current directly enters the heart.
   ii. Is a potentially harmful condition in which a relatively minor part of current injected in the body enters the heart.
   iii. Is the major cause of cardiac arrest in the US.
   iv. Can be prevented by proper active grounding using a driven right leg.
   v. All of the above.
(i) [8 pts] Indicate for each statement below whether it is true or false:

i. **TRUE / FALSE**: The linear variable differential transformer (LVDF) is a highly sensitive inductive sensor.

ii. **TRUE / FALSE**: The transformer in a DC power supply efficiently converts power from AC to DC.

iii. **TRUE / FALSE**: The current dipole generates an electric field that rolls off with the square of distance with an approximate cosine angular dependence.

iv. **TRUE / FALSE**: The Severinghaus electrode is an amperometric sensor measuring partial pressure of carbon dioxide.

v. **TRUE / FALSE**: The IMFET is a special type of ISFET in which ionic charge buildup on the gate results from antigen-antibody binding.

vi. **TRUE / FALSE**: Beer’s law of volume absorbance expresses effects of molar density and optical path length in light propagation through biological media.

vii. **TRUE / FALSE**: The Rheobase current is defined as the threshold for perception for very fast transient stimulation currents.

viii. **TRUE / FALSE**: The third prong in an AC power plug is critically important for safety because it carries the return current of the supplied AC power.
2. **[12 pts]** Analyze the voltage-in, current-out circuit shown below. You may assume the opamp is ideal, and all resistance values have zero error tolerance.

(a) **[6 pts]** Find the output current \( I_{\text{out}} \) as a function of the input voltage \( V_{\text{in}} \). **Hint:** You may assume that this circuit has net negative feedback. It also helps to assume an ideal load at the current output.
(b) [3 pt] Find the input impedance.

(c) [3 pt] Find the output impedance.
3. [12 pts] Now analyze the following current-in, voltage-out transimpedance amplifier circuit, again with ideal opamp and zero-tolerance passive components:

(a) [1 pts] Find the input impedance $Z_{in}$ at the $I_{in}$ node.

(b) [1 pts] Find the output impedance $Z_{out}$ at the $V_{out}$ node.

(c) [4 pt] Find the transfer function $Z_{trans}(j\omega) = V_{out}(j\omega)/I_{in}(j\omega)$. This is by definition the transimpedance of the amplifier.
(d) [6 pt] Sketch the Bode plot of the transimpedance $Z_{\text{trans}}$ from 0.1 Hz to 10 kHz, for $R_3 = 47 \, \text{k}\Omega$ and $C_3 = 100 \, \text{nF}$. Be sure to indicate all units.
4. [10 pts] A Wheatstone bridge with a strain gauge as shown below is used as a transducer to measure stress $\sigma$ as an output voltage $V_o$. The strain gauge has Young’s modulus $E$, gauge factor $G$, and nominal resistance $R_G$. The other resistances $R_1$, $R_2$, and $R_3$ in the bridge are constant, and nominally equal to $R_G$ so the bridge is balanced at zero stress.

\[ V_o = \frac{R_3}{R_3 + R_G} \cdot V \]

(a) [3 pts] Find the output voltage $V_o$ as a function of stress $\sigma$. 

\[ V_o = \frac{R_3}{R_3 + R_G} \cdot V \]
(b) [4 pt] Show that the sensitivity depends on the stress $\sigma$. Find the maximum sensitivity, and the value of $\sigma$ where the maximum is reached.

(c) [3 pt] Now consider a modified version of the transducer circuit below, that uses an opamp with the same strain gauge and same passive components. Show that it offers improved constant sensitivity.
5. **[8 pts]** A cardiologist’s 6-lead frontal ECG machine is partially broken and only displays leads I and aVF. You cleverly observe that this is sufficient to reconstruct the other four leads.

![ECG Diagram]

(a) **[4 pts]** Write leads II, III, aVR, and aVL as a function of your observed leads I and aVF.
(b) [4 pts] 6-lead frontal ECG waveforms from a healthy patient are shown below. Estimate, and sketch on the Einthoven’s triangles below, the direction and relative magnitude of the cardiac vectors $\mathbf{M}$ at the ECG Q and S waves. Show your reasoning. *Hint:* Try to look for leads that are approximately parallel and orthogonal to the cardiac vector.
6. **[8 pts]** Design a non-inverting hysteretic comparator with thresholds 1.4 V and 1.6 V, that draws no more than 100 μA from the input. Include also short-circuit protection at the output, drawing no more than 10 mA from an accidental short. You may use an LM311 device, 3 V battery, and any combination of resistors.
7. [12 pts] Cardiovascular disease is a leading cause of preventable death worldwide. Knowing this, a doctor (who happens to have a degree in bioengineering) is doing tests on patients to assess their cardiovascular health.

(a) [3 pts] The doctor uses a fluid-filled catheter tube pressure measurement system shown below, with fluid inertia $L = 5 \text{ kg/m}^4$, fluid resistance $R = 8 \text{ kg/m}^4\text{s}$, and diaphragm compliance $C = 3 \text{ m}^4\text{s}^2/\text{kg}$. From first principles, find the transfer function $H(j\omega)$ of this system.

(b) [3 pts] Find the damping ratio of the system in part a.
(c) [3 pts] The doctor uses a Hall effect flowmeter to measure the volumetric flow through a blood vessel. The flowmeter induces a magnetic field across the vessel and uses a voltmeter to measure the voltage across the vessel in the direction normal to both magnetic field and blood flow. It also uses calipers to measure the vessel diameter. The system calculates the flow rate to be $100\, \text{cm}^3/\text{s}$ using the known magnetic field, the measured voltage and the measured vessel diameter. After the measurement, the doctor performs a calibration and discovers that the system’s voltmeter was measuring voltages 40% larger than the actual value and the calipers were measuring diameters 10% smaller than the actual value. What was the actual flow rate through the vessel?
(d) [3 pts] The doctor now uses a Doppler effect flowmeter to measure the velocity through the same vessel to validate the calculation from part c. The flowmeter, shown below, transmits a 1 MHz signal from the source and $\theta_r = \theta_s = 45^\circ$. Assuming that the flow rate is the same as calculated in part c and the actual vessel diameter is 5 mm, what frequency do you expect to measure at the receiver?
8. **[8 pts]** A blood pH reader is shown below, with one Ag/AgCl electrode immersed in the blood sample at room temperature, and a second Ag/AgCl electrode inside a glass bulb containing a solution of 0.1 mmol/L HCl in water. The glass membrane is permeable to $H^+$ only. For each Ag/AgCl electrode, the half cell potential is $E_{hc} = 0.223 \text{ V}$ and the impedance parameters are $R_d = 320 \text{ k}\Omega$, $C_d = 1 \text{ pF}$, and $R_s = 1 \text{ k}\Omega$.

![Diagram of pH reader](image)

(a) **[2 pts]** Find the pH of the HCl solution inside the glass bulb.

(b) **[2 pts]** The voltage on the glass electrode relative to the reference electrode measures -186 mV. Find the pH of the blood sample.
(c) [3 pts] Assuming the electrical resistance across the thin glass membrane is $R_{\text{mem}} = 260 \, \text{k}\Omega$, and neglecting the resistance in ionic solution, find the equivalent resistance between the two electrodes at high frequencies.

(d) [1 pt] What happens when the Ag/AgCl electrode in the glass bulb is replaced with a camomel (Hg$_2$Cl$_2$) electrode? Explain.
9. [10 pts] Consider the bioinstrumentation amplifier and driven right leg (DRL) system below with resistances $R_a = 10\, \text{k}\Omega$, $R_b = 1\, \text{k}\Omega$, and $R_f = 2\, \text{M}\Omega$, and with electrode resistances $R_{RA} = 90\, \text{k}\Omega$, $R_{LA} = 110\, \text{k}\Omega$, and $R_{RL} = 100\, \text{k}\Omega$.

(a) [3 pts] Find the effective resistance between the RL terminal and ground.
(b) [6 pts] Find the differential gain $A_d$, common-mode gain $A_c$, and common-mode rejection ratio CMRR.

(c) [1 pts] Explain the importance of $R_o$. 
10. [4 pts] Guest lectures

(a) *Dry-electrode EEG systems*

i. [1 pt] What are the key advantage and greatest challenge in the use of dry electrodes for EEG recording?

ii. [1 pt] List one facial muscle activity that is most strongly observed as artifact in the EEG signal.

(b) *Global health and entrepreneurship*

i. [1 pt] Which country has the highest healthcare costs as fraction of the GDP? List one contributing factor to skyrocketing costs.

ii. [1 pt] Give one guiding principle in the design of medical devices for effective clinical use.