1. An automated sphygmomanometer tightens around a patient’s arm at a pressure of 250 mmHg, and gradually relieves pressure at a rate of 12 mmHg/sec. An microphone in the cuff records the onset of Korotkoff sounds after 9.6 seconds, and the sounds end after 16.5 seconds.

(a) What are the systolic and diastolic blood pressure values?

(b) Determine for each pressure value, whether it is high, low, or normal. If you find some values are abnormal, suggest a possible cause for the abnormalities. Cite any sources you used.

20 points
(1) a) 
\[
\begin{align*}
\text{Systolic: } & \quad 250 \text{ mmHg} - (12 \text{ mmHg/s})(9.6 \text{ s}) = 134.8 \text{ mmHg} \\
\text{Diastolic: } & \quad 250 \text{ mmHg} - (12 \text{ mmHg/s})(16.5 \text{ s}) = 52 \text{ mmHg}
\end{align*}
\]

b) Systolic blood pressure is slightly elevated. Diastolic blood pressure is quite low. Isolated systolic hypertension combined with a lowered diastolic pressure means the pulse pressure (systolic - diastolic) is widened. This could indicate atherosclerotic stiffening of the aorta and risk for cardiovascular disease.

5. Other reasonable answers also acceptable!

2. A magnetic field of 0.5 T is induced across a blood vessel in the positive $z$ direction ($\phi = 0$ in spherical coordinates). The blood vessel has a diameter of 2 mm and is oriented along direction $\theta = 30^\circ$ and $\phi = 100^\circ$ in spherical coordinates. Blood flows in the vessel along this direction.

(a) In what direction does the induced electric field point?

(b) If the voltage across the diameter of the vessel in this direction is measured as 0.9 mV, what is the volumetric flow rate of blood through the vessel?

25 points
(2) a) \[ \phi = 100^\circ = 10^\circ \text{ below horizontal} \]

\[ \vec{E} = -\vec{\nabla} \times \vec{B} \]

\[ \vec{E} = -\vec{\nabla} \times \vec{B} = \vec{B} \times \vec{\nabla} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 0 & -1 \\ \sin \phi \cos \theta & \sin \phi \sin \theta & \cos \phi \end{vmatrix} \]

\[ \vec{E} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 0 & -1 \\ \sin 100^\circ \cos 30^\circ & \sin 100^\circ \sin 30^\circ & \cos 100^\circ \end{vmatrix} \]

\[ = -\sin 100^\circ \sin 30^\circ \hat{i} + \sin 100^\circ \cos 30^\circ \hat{j} = -0.4924 \hat{i} + 0.8529 \hat{j} \]

\[ \rightarrow \text{Divide by } \sqrt{(-0.4924)^2 + (0.8529)^2} \text{ to normalize to a unit vector} \]

Direction: \[ \hat{E} = -0.5 \hat{i} + 0.866 \hat{j} \]

Alternatively in spherical coordinates:

\[ \phi = 90^\circ, \ \theta = 120^\circ \]
2 b) \[ \mathbf{E} = -\mathbf{v} \times \mathbf{B} \]

\[ |\mathbf{E}| = \frac{|\mathbf{v}| |\mathbf{B}| \sin \phi}{
\text{where } \phi = 100^\circ
\]

→ First, find \(|\mathbf{E}|\):

\[ |\mathbf{E}| = \mathbf{v} = \frac{0.9 \text{ mV}}{2 \text{ mm}} = 0.45 \text{ V/m} \]

→ Next, find \(|\mathbf{v}|\):

\[ |\mathbf{v}| = \frac{|\mathbf{E}|}{|\mathbf{B}| \sin \phi} = \frac{0.45 \text{ V/m}}{(0.5 \text{ T}) |\sin(100^\circ)|} = 0.9139 \text{ m/s} \]

→ Next find volumetric flow \(\mathbf{V}\)

\[ \mathbf{V} = A \mathbf{v} = \pi r^2 \mathbf{v} = \pi \left(1 \times 10^{-2} \text{ m}\right)^2 (0.9139 \text{ m/s}) \]

\[ = 2.87 \times 10^{-6} \text{ m}^3/\text{s} \quad \text{convert } \left(\frac{100 \text{ cm}}{\text{m}}\right)^3 \]

\[ = 2.87 \text{ mL/s} \]
3. The Doppler effect can be applied to achieve noninvasive blood flow measurement with high precision. Consider a simplified model of Doppler blood flow measurement with a single blood vessel, an ultrasonic transmitter, and an ultrasonic receiver placed either at positions A, B, or C on the body as shown below. The diameter of the blood vessel \( d \) is 3 mm, the frequency of the ultrasound emitted from the transmitter is 1 MHz, and the speed of sound in body tissue \( c = 1,500 \text{ m/s} \).

**20 points**

(a) The receiver located at position A observes the signal with frequency 1 MHz + 188 Hz. What is the volumetric flow rate of blood in the vessel?

(b) Now moving the location of the receiver, find the frequency of the signal observed when the receiver is in position B, and when it is in position C. In which of the three locations A, B, and C do you obtain the highest sensitivity to blood flow velocity?
Problem 3:

Part (a):

\[ f_{R-A} = f_s \cdot \left( 1 + \frac{V}{c} \cdot \cos \theta_s \right) \cdot \left( 1 + \frac{V}{c} \cdot \cos \theta_R \right) \]

\[ = f_s \left( 1 + 0 \right) \cdot \left( 1 + \frac{V}{c} \right) \]

\[ = 1 \text{ MHz} \times \left( 1 + \frac{\sqrt{2}}{1500 \text{ m/s}} \right) = 1 \text{ MHz} + 188 \text{ Hz} \]

\[ \Rightarrow V = 0.282 \text{ m/s} \quad \text{flow velocity} \]

Volumetric flow rate = \( v \cdot A = 1.99 \times 10^{-6} \text{ m}^3/\text{s} \)

Part (b):

\[ f_{R-B} = f_s \left( 1 + \frac{V}{c} \cdot \cos \theta_s^{\circ} \right) \cdot \left( 1 + \frac{V}{c} \cdot \cos \theta_R^{\circ} \right) \]

\[ = f_s = 1 \text{ MHz} \]
\[ \text{Sensitivity} \bigg|_{-B} = \left| \frac{d f_{R-B}}{d V} \right| = 0 \]

\[ f_{R-C} = f_s \left( 1 + \frac{V}{C} \cos 90^\circ \right) \left( 1 + \frac{V}{C} \cos 180^\circ \right) \]

\[ = f_s \left( 1 - \frac{V}{C} \right) = f_s - \frac{f_s}{C} \cdot V \]

\[ \text{Sensitivity} \bigg|_{-C} = \left| \frac{d f_{R-C}}{d V} \right| = \frac{f_s}{C} \]

As shown in part a.

\[ f_{R-A} = f_s + \frac{f_s}{C} \cdot V \]

\[ \text{Sensitivity} \bigg|_{-A} = \frac{f_s}{C} \]

Therefore, you will get same sensitivity at position A and B.
4. **Design Problem:** 35 points

Design a system for measuring intravascular blood pressure, transducing this pressure at its input to a voltage at its output, using the following components:

(a) a fluid-filled catheter of at least 30 cm in length, that is critical-damped and operates in the frequency range from 0 to 30 Hz;

(b) a fluid chamber between the catheter and a membrane registering pressure as strain with Young’s modulus 30 kPa;

(c) a strain gauge with nominal resistance $R_G = 10 \, \text{k} \Omega$ at zero strain, and with gauge factor $G = 1$.

(d) a Wheatstone bridge circuit to convert pressure to voltage;

and the following specifications:

(a) Use a single 3V battery.

(b) The target sensitivity is 10 mV/mmHg.

(c) The operating range of blood pressure is 0-200 mmHg.

(d) When the pressure reaches higher than 150 mmHg, then an alert should be triggered by lighting a red LED.

As always, be sure to show a complete diagram of your design, and specify all components including the types and values where applicable, including, for instance, the internal diameter of the catheter and its length. Provide reasoning for justifying your design choices, and state all assumptions supporting the reasoning. You may use the second-order circuit analog to the catheter-membrane-strain gauge system that we studied in Lecture 13.

10 points for r, 10 points for sensitivity, 5 points for alert function, 5 points for wheatstone bridge, 5 points for other requirements.
Design problem:

Based on Lecture 13:

\[ \frac{P_{\text{in}}}{V_{\text{in}}} \xrightarrow{R} \frac{P_{\text{out}}}{V_{\text{out}}} \]

\[ C_{d} \xrightarrow{=} \]

\[ \begin{cases} P_{\text{in}} = V_{\text{in}} \\ P_{\text{out}} = V_{\text{out}} \end{cases} \]

\[ \frac{P_{\text{out}}(j\omega)}{P_{\text{in}}(j\omega)} = \frac{1}{1 + R C_{d} j\omega - L C_{d} \omega^2} \]

\[ = \frac{1}{1 + \frac{2 R C_{d} j\omega}{W_n} - \frac{\omega^2}{W_n^2}} \]

With

\[ W_n = \frac{1}{\sqrt{L C_{d}}} > 2\pi \times 30 \]

\[ R \geq \frac{R C_{d}}{W_n} \]
\[ z = \frac{R}{2} \sqrt{\frac{C_d}{2}} = \frac{R \cdot C_d \cdot W_n}{2} = 1 \] (Critical-damped)

\[ \frac{\Delta V}{V} = 3 \cdot \frac{\Delta l}{l} \approx 3 \cdot \frac{\Delta l}{l} = 3 \varepsilon \]

\[ \Rightarrow C_d = \frac{1}{E_d} = \frac{\Delta V}{\Delta p} = 3V \cdot \frac{\Delta l}{\Delta p} = 3V \cdot \frac{\Delta l}{\Delta p} = 3V \cdot \frac{1}{E_Y} \]

\[ C_d = 3V \times \frac{1}{30 \times 10^3 \text{Pa}} \]

\[ \left\{ \begin{array}{l}
W_n = \frac{1}{\sqrt{L \cdot C_d}} \geq 60 \varepsilon \\
L < \frac{1}{C_d \cdot (60 \varepsilon)^3}
\end{array} \right. \]

\[ \Rightarrow \left\{ \begin{array}{l}
R \cdot C_d \cdot W_n = \frac{1}{2} \\
R^2 \cdot C_d = 4L
\end{array} \right. \]

\[ \left\{ \begin{array}{l}
R = \frac{8 \pi \cdot l}{\pi \cdot l^4} \\
L = \rho \cdot \frac{1}{\pi \cdot l^2} \\
R^2 \cdot C_d = 4L
\end{array} \right. \]

\[ \Rightarrow \left\{ \begin{array}{l}
l < \frac{\pi r^2}{C_d \cdot \rho \cdot (60 \varepsilon)^3} \\
64 \pi^2 l^2 \cdot C_d = \frac{4 \pi l}{\pi r^2}
\end{array} \right. \]
Assume the blood viscosity is \(3.5 \times 10^{-3} \text{ Pa.s}\) and the blood mass density is \(1060 \text{ Kg/m}^3\).

\[
\begin{align*}
\Rightarrow & \quad 0.3 \text{ m} < \frac{\rho \pi r^6}{16 \eta^2 \cdot C_d} < \frac{\pi r^2}{C_d \cdot (60 \pi)^2} \\
\Rightarrow & \quad \frac{4.8 \eta^2 C_d}{\rho \pi} < r^6 < \frac{16 \eta^2 \cdot r^2}{\rho^2 \cdot 60^2 \pi^2}
\end{align*}
\]

\[
0.000177 C_d \times 10^{-6} < r^6 < 49.145 \times 10^{-6} \cdot r^2
\]

\[
(0.000177 C_d)^{\frac{1}{6}} \times 10^{-1} < r < 2.65 \times 10^{-4} \text{ (m)}
\]
As: \( C_d = 3V \times \frac{1}{30 \times 10^3 \text{Pa}} \)

and make: \( R = 25 \times 10^{-4} \text{m} = 250 \mu \text{m} \)

\[ \Rightarrow (0.0177 \times 3V \times \frac{1}{30 \times 10^3 \text{Pa}})^\frac{1}{6} \times 10^{-7} < 2.5 \times 10^{-4} \]

\[ \Rightarrow V < 1.38 \times 10^{-10} \text{ m}^3 \]

Set \( V = 0.138 \text{ mm}^3 \)

\[ E = \frac{\Delta P}{E} \]

\( 1 \text{ mm Hg} = 133.3 \text{ Pa} \)

\( E = 30 \text{ kPa} = 225 \text{ mm Hg} \)

\( 0 \leq P \leq 200 \text{ mm Hg} \Rightarrow 0 \leq 3E \leq 0.8889 \)

\( R^* = R_G (1 + GE) \)

\( 10 \text{ kN} \leq R^* \leq 18.889 \text{ kN} \)
You can also use Wheatstone bridge for first stage.

\[
\begin{align*}
V_{\text{out}} &= \frac{V}{R_3} \cdot R_G + V_-
\end{align*}
\]

\[
V_- = V_+ = \frac{R_2}{R_1 + R_2} \cdot 3 \ (V)
\]

\[
V_{\text{out}} = \frac{V}{R_3} \cdot R_G \left(1 + G \cdot \frac{P_{\text{out}}}{\ell} \right) + V
\]

As the gain of fluid-filled catheter is 1,
If it operates in range from 0 to 30Hz.

So, \( P_{\text{out}} = P_{\text{in}} \)

\[
V_{\text{out}} = \frac{V_\text{in}}{R_2} \cdot R_G \left( 1 + G \cdot \frac{P_{\text{in}}}{E} \right) + V_\text{in}
\]

To meet the requirements:

\[
V_{\text{out, maximum}} = \frac{V_\text{in}}{R_2} \cdot R_G \left( 1 + G \cdot \frac{200\text{mmHg}}{E} \right) + V_\text{in} \leq 3\text{V}
\]

\[
\begin{align*}
\text{Sensitivity} = \frac{dV_{\text{out}}}{dP_{\text{in}}} &= \frac{V_\text{in}}{R_3} \cdot R_G \cdot G \cdot \frac{1}{E} = \frac{1\text{mV}}{\text{mmHg}}
\end{align*}
\]

\[
\Rightarrow 2.25 \left( 1 + \frac{200}{2 \times 10^5} \right) + \frac{R_3}{10 \times 10^3} \times 2.25 \leq 3
\]

\( R_3 \) is a negative value!

So we cannot use single stage amplifier to get the sensitivity. We need at least one more stage amplifier, and lower gain.
for first stage!

\[ V_{\text{out}} = \frac{V}{R_5} \cdot R_G \left( H \cdot G \cdot \frac{200 \text{ mmHg}}{\varepsilon} \right) + V \leq 3 \text{ V} \]

\[ \text{Sensitivity} = \frac{dV_o}{dP_{\text{in}}} = \frac{V}{R_3} \cdot R_G \cdot G \cdot \frac{1}{\varepsilon} = 4 \text{ mV/mmHg} \]

\[ \frac{V}{R_3} \times 10^4 \times \frac{1}{24} = 0.004 \]

Choose \[ \begin{cases} R_3 = 10^4 \\ V = 0.9 \text{ V} \end{cases} \implies V_4 = \frac{R_2}{R_1 + R_2} \times 3 = 0.9 \text{ V} \]
Choose the following values:

\[ V_2 = 0.9 V \quad \Rightarrow \quad V_4 = R_1 + R_2 \approx 1 V \]

\[ R_2 = 300 \, k \]

\[ R_1 = 700 \, k \]

Choose high values for \( R_1 \) and \( R_2 \) to reduce power consumption!

\[ A = \frac{R_5}{R_4 + R_5} \leq 1 \]

\[ \frac{V_{out} - A \cdot V_6}{R_7} = \frac{A V_6 - V_2}{R_6} \]

\[ V_{out} = \frac{R_7}{R_6} (A V_6 - V_2) + A \cdot V_6 \]

\[ V_{out} = \left( \frac{R_7}{R_6} + 1 \right) \cdot A \cdot V_6 - \frac{R_7}{R_6} \cdot V_2 \]

To get 10 mV/MMHg sensitivity, the gain of the second stage should be 2.1
\[(\frac{R_5}{R_b} + 1)A = 2.5\]

1.8V \leq V_0 \leq 2.6V \implies 4.5V \leq 2.5V_0 \leq 6.5V

\[\text{again, to avoid saturation, } 1V \leq V_{\text{out}} \leq 3V\]

\[\Rightarrow 3V \leq \frac{R_7}{R_b} \cdot V_2 \leq 4.5V\]

\[\text{to keep circuit simple, let's make } A = 1\]

\[R_5 = \infty, \text{ so just remove } R_5,\]

\[\Rightarrow \frac{R_7}{R_b} = 1.5 \implies V_2 = 3\ \text{V}.\]

\[
\begin{align*}
R_7 &= 150k \\
R_6 &= 100k
\end{align*}
\]

or other reasonable values.

so. when \(P_{in} = 0 \text{ mV}\), \(V_{\text{out}} = 0 \text{V}\).
\[ P_{in} = 150 \text{ mm Hg}, \quad V_{out} = 1.5 \text{ V} \]

\[ P_{in} = 200 \text{ mm Hg}, \quad V_{out} = 2 \text{ V} \]

If you make \( 2V < V_e < 3V \), you can use the following circuit to get it, you need

\[ \frac{3V}{2} \]

\[ \frac{R_8}{2} \quad \frac{R_9}{2} \]

Relative small values for \( R_8, R_9 \) to reduce load effect of \( R_6 \).

Now, the sensitivity of whole system is 10 mV/mm Hg, and only +3 V power supply is used.

Next, let's add the alarm function.
Similarly, let's make \[
\begin{align*}
R_{10} &= 500k \\
R_{11} &= 500k
\end{align*}
\]
So far, we already meet all the requirements.

If you are interested, please read the following contents.

As we just assumed:

\[ W_n = \text{cut-off frequency}, \]

actually, by definition.

cut-off frequency is

\[ |H(j\omega)| = \frac{1}{\sqrt{2}} \]

\[
\begin{align*}
\frac{P_{out}(j\omega)}{P_{in}(j\omega)} &= \frac{1}{(1 + \frac{j\omega}{W_n})^2} \\
H(j\omega)| &= \left| \frac{P_{out}(j\omega)}{P_{in}(j\omega)} \right| = \frac{1}{H \left( \frac{W_n}{W_n} \right)^2} = \frac{1}{\sqrt{H(j\omega_1)^2} \sqrt{H(j\omega_2)^2}}
\end{align*}
\]
\[ |H(j\omega)| = \frac{1}{\sqrt{2}} = \frac{1}{1 + \left(\frac{W_c}{\omega_n}\right)^2} \]

Now let's figure out cut off frequency \( W_c \).

As the fluid-filled catheter needs to operate in the frequency range from 0 to 30 Hz,

So, the cut-off frequency \( W_c > 2\pi \times 30 \)

By definition:

\[ |H(jW_c)| = \frac{1}{\sqrt{2}} = \frac{1}{1 + \left(\frac{W_c}{\omega_n}\right)^2} \]

\[ \Rightarrow \left(\frac{W_c}{\omega_n}\right)^2 = \sqrt{2} \]

\[ \Rightarrow \begin{cases} 
\left(\frac{W_c}{\omega_n}\right)^2 = \sqrt{2} \\
W_c > 60\pi 
\end{cases} \Rightarrow \omega_n > 29.3 \]