1. As a BENG 199 intern in a biochemistry lab on campus you are preparing an experiment for which you need access to a fume hood. The fume hood available to you in the lab has a control panel for the exhaust fan with three mysterious switches. The control panel was designed by a former lab mate who since left for graduate school elsewhere. The control manual for the fume hood is missing, but luckily you find a copy of the circuit schematic below. You decide to carefully study the circuit schematic and the printed circuit board before boldly exploring its function by trying the different states of the switches $S$, $d_0$ and $d_1$ on the panel. In addition to the switches, the printed circuit circuit has a 3V supply, a 555 timer IC, another IC with four NAND logic gates, and passive components $R = 1 \, \text{k}\Omega$, $R_1 = 1 \, \text{M}\Omega$, $C = 10 \, \text{nF}$, $C_1 = 10 \, \mu\text{F}$, and $C_2 = 100 \, \mu\text{F}$.

(a) You observe that no matter the state of the binary input control switches $d_0$ and $d_1$, the exhaust fan stays off when the pushbutton switch $S$ is untouched (left in the OFF position). You also observe that when $d_0$ is in the binary 0 state, whenever the pushbutton $S$ is pressed (goes briefly in the ON position) the exhaust fan goes on for some time, and then goes off until $S$ is pressed again. The time duration of the fan blowing depends on the state of $d_1$. Explain why this happens, and find the two time durations for both binary states of the $d_1$ switch, 0 and 1.

(b) You find that when $d_0$ is in the binary 1 state, the exhaust fan goes on for a very brief time (about 50 ms) when the pushbutton $S$ is pressed, regardless of the binary state of $d_1$. Why might this be happening?
(c) Can you figure a possible way how the logic for the $d_0$ and $d_1$ control in the circuit schematic was implemented with the four NAND gates of the logic IC on the printed circuit board?

(d) **Design Problem:** Now modify the design of the circuit to provide control, using the same switches $S$, $d_0$ and $d_1$ on the panel, to select between one of four time durations for the exhaust fan: 20 s, 70 s, 100 s, and 150 s. As before, pressing the pushbutton $S$ should activate the exhaust fan, and the binary states of $d_0$ and $d_1$ should control the time duration. *Hint:* you can (but don’t need to!) do it using three capacitors without the NAND gates or other control logic.
part (a):

When switch S is untouched, voltage of pin 2 (Trigger signal input) is high. According to the character of 555 timer, Out (pin 3) will be always low. So the fan stays off.

While switch S is pressed, voltage of pin 2 (Trigger signal input) is low. Then voltage of Out (pin 3) goes high, and keeps high until THR (pin 6) goes higher than CTRL (pin 5) which is \(2V_{cc}/3\). Here the voltage THR is determined by an RC circuit, namely

\[ \text{THR} = V_{cc}(1 - \exp(-t/RC)). \]

Time constant \(RC\) will be either \(R1*C1\) or \(R1*C2\), which depends on the combination of \(d0\) and \(d1\), as \(d0\) and \(d1\) control the output of two AND gates.

When \(d0=0\) and \(d1=1\), \(C1\) will be selected, then

\[ \text{THR} = V_{cc}(1 - \exp(-t/R1*C1)) = 2V_{cc}/3 \]

\[ t = R1*C1*\ln(3) = 10.986 \text{ (s)} \]

When \(d0=0\) and \(d1=0\), \(C2\) will be selected, then

\[ \text{THR} = V_{cc}(1 - \exp(-t/R1*C2)) = 2V_{cc}/3 \]

\[ t = R1*C2*\ln(3) = 109.86 \text{ (s)} \]

(b): When \(d0=1\), the output of two AND gates will always be low, no matter what the binary state of \(d1\) is. Then no capacitor will be selected. THR will always be high (higher than \(2V_{cc}/3\)), as which is connect to \(V_{cc}\) via \(R1\).

When the pushbutton S is pressed, pin 2 (Trigger signal input) goes from High to low. A falling edge will be generated, and this falling edge will make Out goes from low to high. But THR is higher than \(2V_{cc}/3\), then Out goes from high to low immediately, about 50 ms.

(c):

Use four NOR gates to implement the logic for \(d0\) and \(d1\) in the figure.
The logic is as following: when $d_0=0$, either $C_1$ or $C_2$ can be selected by $d_1$;

when $d_0=1$, neither $C_1$ nor $C_2$ can be selected regardless of binary state of $d_1$.

Circuit 1:

![Circuit Diagram 1]

Circuit 2:

![Circuit Diagram 2]
part (d)

\[
\text{approach 1:}
\]

\[
\begin{align*}
&d_0 = 0, \ d_1 = 0, \quad Z = R_1 C_1 \\
&d_0 = 1, \ d_1 = 0, \quad Z = R_1 (C_1 + C_2) \\
&d_0 = 0, \ d_1 = 1, \quad Z = R_1 (C_1 + C_2) \\
&d_0 = 1, \ d_1 = 1, \quad Z = R_1 (C_1 + C_2 + C_3)
\end{align*}
\]

\[\Rightarrow\]
\[
\begin{align*}
C_1 &= 20 \\
C_2 &= 50 \\
C_3 &= 80
\end{align*}
\]

Choose the unit you prefer, then choose \( R_1 \).
Approach 2:

\[
\begin{align*}
R_1 C_1 \ln(3) &= 20 \text{ (s)} \\
R_1 C_2 \ln(3) &= 70 \text{ (s)} \\
R_1 C_3 \ln(3) &= 100 \text{ (s)} \\
R_1 C_4 \ln(3) &= 150 \text{ (s)} \\
R_1 &= 1\text{M ohms} \\
C_1 &= 18.2 \text{ uF} \\
C_2 &= 63.7 \text{ uF} \\
C_3 &= 91.0 \text{ uF} \\
C_4 &= 136.5 \text{ uF}
\end{align*}
\]
2. Next term you are doing an internship in a neuroengineering lab, working on miniaturized instrumentation for DNA detection and analysis. The design of the instrument uses a light-emitting diode (LED) light source to excite a specimen and produce fluorescence in a DNA microarray. For this purpose you need to pulse the LED at a frequency of 1 MHz. However the crystal oscillator that fits the form factor of the instrument only provides a 500 kHz pulse frequency. You decide to implement the frequency-doubler circuit below (a) to up-convert the 500 kHz input signal from the crystal oscillator to a 1 MHz output signal driving the LED. The waveform from the crystal oscillator at the input of the circuit is shown in (b) below.

(a) Sketch the voltage waveforms of the circuit at its input (Input), at the inverting input of the comparator $C_1$, at the comparator output $V_0$, and at the XOR output (Output).

(b) Find suitable values for $R_1$ and $C_1$ such that for the 500 kHz input the duty cycle of the 1 MHz output waveform is near 50%.

(c) Show that the circuit operates as a frequency-doubler over a wide range of input frequencies.
part (a) When time constant $R_1C_1$ is small:
part (a)

When time constant $R_1C_1$ is large:
\[
(3 - (1.5 - a)) \left(1 - e^{-\frac{0.5\mu s}{T}}\right) = (3 - 1.5) \left(1 - e^{-\frac{0.5\mu s}{T}}\right) = a
\]
\[
(3 - (1.5 - a)) \left( 1 + e^{-\frac{0.495 \text{ ms}}{T}} \right) = (3 - 1.5) \left( 1 - e^{-\frac{0.5 \text{ ms}}{T}} \right) = a
\]

\[
(1.5 + a) \times \frac{0.495 \text{ ms}}{T} = 1.5 \times \frac{0.5 \text{ ms}}{T} = a
\]

*a doesn't exist!*

So the duty cycle won't be exact 50%.

Let's make duty cycle 51%:

\[
\begin{align*}
\text{Then:} & \\
(3 - (1.5 - a)) \left( 1 + e^{-\frac{0.495 \text{ ms}}{T}} \right) = (3 - 1.5) \left( 1 - e^{-\frac{0.5 \text{ ms}}{T}} \right) = a \\
(1.5 + a) \times \frac{0.495 \text{ ms}}{T} = 1.5 \times \frac{0.5 \text{ ms}}{T} = a
\end{align*}
\]

\[
\begin{cases}
  a = 0.0612 \\
  T = 12.5 \text{ ms}
\end{cases} \Rightarrow R_1, C_1 = 12.5 \text{ ms}
\]
They you can choose basically any combination of \( R_1, C_1 \). For example:
\[
\begin{aligned}
  & R_1 = 12.5 \text{k} \\
  & C_1 = 1 \text{nF}
\end{aligned}
\]

Part (c)

R1 and C1 function a low pass filter, then the average of the voltage of C1 will be 1.5, and the voltage will always pass through the 1.5 V threshold, like the two situations shown on the right side.

If the comparator is ideal, then this circuit can double the frequency of any input.
3. Given below is an active sensor circuit for precise measurement of body temperature. The circuit combines a Wheatstone bridge with an active output stage implementing an inverting amplifier. $R_3$ is a thermistor with transfer function

$$R_3(T) = R_\infty \exp\left(\frac{\beta}{T}\right)$$

where $T$ is the absolute temperature, $\beta = 4,219$ K, and $R_\infty = 0.0123$ $\Omega$. The passive components are sized $R_1 = R_2 = R_4 = R = 10$ k$\Omega$, and $R_5 = 100$ k$\Omega$. The Wheatstone reference voltage is $+V = 1$ V.

(a) Show that at nominal body temperature (310 K) the Wheatstone bridge is balanced (all resistances are equal).

(b) Find the voltage output $V_{out}$ as a function of temperature $T$, and plot the result over the typical range of body temperatures. What is it at body temperature?

(c) Find the sensitivity at nominal body temperature.

(d) What are the advantages of using the Wheatstone bridge, and using the active output stage?
**Problem 3**

*a)\[ R_3 (310) = 0.0123 \exp \left( \frac{4219}{310} \right) = 10011.8 \approx 10K\Omega \]

Therefore, the Wheatstone bridge is balanced as all resistances are equal.

*b)\[
\begin{aligned}
\frac{V - V_{A}}{R_3} &= \frac{V_{A}}{R_4} - \frac{V_{0} - V_{A}}{R_5} \\
1 - 0.5 &= 0.5 - \frac{V_{0} - 0.5}{10K\Omega} - \frac{V_{0} - 0.5}{100K\Omega}
\end{aligned}
\]

\[ V_0 = (100,000) \left( \frac{0.5 - 1 - 0.5}{10,000} + \frac{0.5}{R_3} \right) \]

\[ V_0 = 0.5 + 5 - \frac{50,000}{R_3} \]

\[ V_0 = 5.5 - \frac{50,000}{0.0123 \exp \left( \frac{4219}{T} \right)} \]
Plot (+1.5)

\[ V \]

\[ V_{\text{out}}(V) \]

\[ T \]

Temperature (K)

\[ 0.3 \]

\[ 309 \]

\[ 0.59 \]

\[ \approx 313K \]

At body temperature,

\[ V_0(310) = 5.5 - \frac{50000}{\left(0.0123\right)\exp\left(\frac{4219}{310}\right)} = 5.5 - \frac{50000}{10000} = 0.5V \]

\[
\begin{align*}
\text{C) Sensitivity } & = \frac{dV_0}{dT} = \frac{d}{dT} \left[ 5.5 - \frac{50000}{0.0123 \exp\left(\frac{4219}{T}\right)} \right] \\
& = \left( \frac{-50000}{0.0123} \right) \left( \frac{4219}{T^2} \right) \exp\left(\frac{-4219}{T}\right) \\
& \approx -0.22 \text{ V/K}
\end{align*}
\]

\[ \left. \frac{dV_0}{dT} \right|_{310K} = -0.22 \text{ V/K} \]
4. Consider the active circuit below depicting a second-order lowpass filter, known as a Sallen-Key filter.

![Circuit Diagram]

(a) For an ideal opamp ($A_v = \infty$) find the transfer function $H(j\omega) = \frac{V_{out}(j\omega)}{V_{in}(j\omega)}$.

(b) Find the range of DC gains $|H(0)|$, and the range of damping factors $\zeta$, that can be realized with this circuit.

(c) Find suitable values for the components for a critically damped second-order low-pass response with cut-off frequency $f_c = 160$ Hz and DC gain of 20 dB.

(d) Sketch the Bode plot of this frequency response.
d) Wheatstone bridge - high sensitivity, variable input voltage
Active output circuit - amplification of signal

Problem 4

\[ Av = \infty, \text{ Find } H(j\omega) = \frac{V_{out}(j\omega)}{V_{in}(j\omega)} \]

Applying KCL at A, \( \frac{V_{in} - V_A}{R_1} = \frac{V_A - V_{out}}{Z_1} + \frac{V_A - V_B}{R_2} \)

where \( Z_1 = \frac{1}{j\omega C_1} \), \( Z_2 = \frac{1}{j\omega C_2} \)

Voltage divider: \( V_B = \frac{Z_2}{Z_2 + R_2} \cdot V_A \)

\[ V_B = \frac{R_B}{R_B + R_A} \cdot V_{out} \Rightarrow V_A \cdot \frac{Z_2}{Z_2 + R_2} = \frac{R_B}{R_B + R_A} \cdot V_{out} \]

\[ \Rightarrow V_A = \frac{R_B(Z_2 + R_2)}{Z_2 + R_2 + R_B} \cdot V_{out} \]
Plugging \( V_A \) back to equation 1,

\[
\frac{V_{in} - \frac{R_B (R_2 + R_2)}{R_2 (R_A + R_B)}}{R_1} = \frac{R_B (R_2 + R_2)}{R_2 (R_A + R_B)} \frac{V_{out} - V_{out}}{Z_1} + \frac{R_B (R_2 + R_2)}{R_2 (R_A + R_B)} \frac{V_{0} - \frac{R_B}{R_A + R_B}}{R_2}
\]

\[\Rightarrow \text{Simplifying, you get:}\]

\[
H(j\omega) = \frac{1}{R_1} \frac{1}{R_1} \frac{R_B}{R_A + R_B} (j\omega R_2 C_2 + 1) + \frac{1}{R_2} \frac{R_B}{R_A + R_B} (j\omega R_2 C_2 + 1) - \frac{1}{R_2} \frac{R_B}{R_A + R_B}
\]

\[\Rightarrow + \frac{R_B}{R_A + R_B} (j\omega R_2 C_2 + 1) \frac{1}{1} C - j\omega C
\]

\[
(b) \quad H(0) = \frac{1}{R_1} \frac{1}{R_1} \frac{R_B}{R_A + R_B} (1) + \frac{1}{R_2} \frac{R_B}{R_A + R_B} (1) = \frac{R_A + R_B}{R_B}
\]

\[
|H(0)| = \frac{R_A}{R_B} + 1
\]

2nd order filter standard form:

\[
H(j\omega) = H(s) = \text{Gain} \cdot \frac{1}{(\frac{s}{W_0})^2 + 2\frac{\zeta}{W_0} s + 1}
\]

Let \( \frac{R_B}{R_A + R_B} = \chi \)

\[
H(s) = \frac{1}{R_1} \frac{1}{R_1} \chi (s R_2 C_2) + \frac{1}{R_2} \chi (s R_2 C_2) \frac{R_A}{R_2} + \chi (s R_2 C_1)
\]

\[\Rightarrow + \chi s C - s C
\]
Simplifying, \( H(s) = \frac{x}{s^2(R_1 R_2 C_2) + s(R_2 C_2 + R_1 C_2 + C_1 - kC_1) + 1} \)

According to the standard form, gain = \( x \cdot \frac{R_1 R_2 C_2}{W_0^2} \)

\( R_{2C_2} + R_1 C_2 + C_1 - xC_1 = \frac{2\pi}{W_0} \)

\( \Rightarrow \frac{2}{2} = \frac{W_0 (R_{2C_2} + R_1 C_2 + C_1 - xC_1)}{2} \)

\[ z = \sqrt{\frac{R_1 R_2 C_2 (R_{2C_2} + R_1 C_2 + C_1 - (\frac{R_A}{R_B} + 1)C_1)}{2}} \]

(3) \( f_c = 160 \text{ Hz} \)

\( \omega_c = 2\pi f_c = 1005.3 \)

DC gain = 20 dB = 20 \log_{10}(1H(0))

\( 20 = 20 \log_{10}(\frac{R_A}{R_B} + 1) \Rightarrow 1 = \log_{10}(\frac{R_A}{R_B} + 1) \)

\( \Rightarrow 10 = \frac{R_A}{R_B} + 1 \)

Critically damped \( \Rightarrow z = 1 \)

\( \therefore H(s) = \frac{(\frac{R_A}{R_B} + 1)}{s^2(R_1 R_2 C_2) + s(\frac{2}{\sqrt{R_1 R_2 C_2}}) + 1} \)

\( W_0 = \omega_c = 1005.3 = \sqrt{R_1 R_2 C_2} \Rightarrow R_1 R_2 C_2 = 1010628.09 \)

\( \therefore \) Possible values:

- \( R_1 = R_2 = 10 \text{ M}\Omega \)
- \( R_A = 900 \text{ k}\Omega \)
- \( C_1 = C_2 = 100 \mu F \)
- \( R_B = 100 \text{ k}\Omega \)