Parts of the homework done collectively in-class (Friday, January 12) are indicated below with [IC].

1. A thermocouple outputs a voltage $V$ that depends on temperature $T$ (in K) as:

$$V(T) = a + bT + cT^2$$

where $a = -30$ V, $b = 0.1$ V/K, and $c = 0.002$ V/K$^2$.

(a) [IC] Find the sensitivity of the voltage output as a function of temperature. What is it at nominal body temperature $T_0 = 310$ K?

(b) [IC] A 10-bit analog-to-digital converter (ADC) produces a digital output $D$. Find the voltage range that is required of the ADC so that the corresponding temperature range spans from 300 K to 320 K.

(c) Estimate the temperature $T$ as a function of the digital output $D$ of the ADC (from 0 to 1023) to be used for a reading display of the temperature sensor.

(d) Find the worst-case accuracy of this temperature reading over the temperature range.

(e) Now consider that the voltage of the thermistor is subject to additive white noise with standard deviation of 50 mV. Find the precision of the temperature reading obtained from a single sample. How many samples do you need to take for each reading in order for the precision of the sample-averaged output to be better than the accuracy? (Hint: think of the Central Limit Theorem.)

2. Consider the current-input, voltage-output circuit below, with $R_1 = 100$ k$\Omega$, $R_2 = 1$ k$\Omega$, $C_2 = 10$ nF, and $C_3 = 1$ $\mu$F.

(a) [IC] What is the impedance of an ideal source $I_{in}$ feeding the current input, and the impedance for an ideal load at the voltage output $V_{out}$?

(b) Find the input impedance $Z_{in}(j\omega)$ of the circuit, and show its Bode plot (log magnitude and phase versus log frequency). (Hint: assume an ideal load at the circuit output.)

(c) [IC] Find the output impedance $Z_{out}(j\omega)$ of the circuit, and show its Bode plot. (Hint: assume an ideal source at the circuit input.)

(d) Find the transfer function $H(j\omega) = V_{out}(j\omega) / I_{in}(j\omega)$, and show its Bode plot.
3. **Design Problem:** In this problem we will design an anti-alias lowpass filter for an electrocardiography (ECG) recording system. The ADC samples the ECG signal at 5 kHz.

   (a) [IC] Find the maximum cut-off frequency for an ideal lowpass filter.

   (b) [IC] Find the transfer function of the anti-alias filter with a second-order lowpass response with natural frequency $f_n = 2$ kHz and damping factor $\zeta = 1.25$.

   (c) What is the corresponding (-3dB) cut-off frequency $f_c$? Is this choice of cut-off frequency reasonable for the particular second-order filter and for typical ECG signals?

   (d) Design a circuit implementing the anti-alias filter using only resistors and capacitors. The input impedance of your filter circuit needs to be at least 1 MΩ over the 0-1 kHz range for ECG signals. Hint: consider a cascade of two RC first-order lowpass sections.

   (e) How close is the voltage acquired by the ADC to the ECG signal at the input of the filter, when the ECG electrode impedance is 100 kΩ? Find the transfer function as a function of frequency.