Midterm

NAME: _SOLUTIONS_

Guidelines:

1. Open book, open notes, open mind. No communication with others.
2. Time limit: 1 hour 15 minutes (in-class).
3. Mark directly on the handout.

The midterm counts for 25% towards the final grade.
Problem 1 (20 Points):

Sketch the cross-section of possible fabricated structures for the following devices in a standard CMOS process. Denote $n$ and $p$ doped regions (including substrate, well and base regions, if any). You may assume $p$-base is available in an $n$-well process, and $n$-base in a $p$-well process. Connect bulk/substrate terminals to GND or Vdd as needed.

1a. An $n$MOS (non-cascoded) current mirror in a $p$-well process (terminals IN and OUT);

![Diagram of nMOS current mirror in p-well process]

1b. A $pnpn$ thyristor in an $n$-well process (terminals P1, N1, P2, N2; all terminals disconnected from the substrate);

![Diagram of pnpn thyristor in n-well process]
1c. An $pnp$ vertical bipolar transistor in a $p$-well process (terminals E, B, C; all terminals disconnected from the substrate);

![Diagram of an $pnp$ vertical bipolar transistor]

1d. A $pn$ photodiode in an $n$-well process (terminals OUT and GND, denote photosensitive region).

![Diagram of a $pn$ photodiode]
Problem 2 (40 Points):

Design a digital CMOS, 2-bit Gray-code counter. Assume you have a non-overlapping two-phase clock $\phi_1, \phi_2$ available, along with their complements. Minimize the number of transistors used. Show the entire transistor-level circuit diagram. You can save ink by using sub-circuit “cells” as long as you clearly specify each of them and their terminals.

*HINT:* the counter could be implemented as sequential logic $Y[n] = f(Y[n-1])$, where the outputs $Y_1[n]Y_0[n]$ over time repeat the sequence 00, 01, 11, 10.

\[ Y_{n-1} \xrightarrow{f} Y_n \]

\[ \begin{array}{c|c|c|c}
   \text{[m-1]} & \text{[m]} \\
   Y_1 & Y_0 & Y_1 & Y_0 \\
   \hline
   0 & 0 & 0 & 1 \\
   0 & 1 & 1 & 1 \\
   1 & 1 & 1 & 0 \\
   1 & 0 & 0 & 0 \\
\end{array} \]

\[ Y_0 \left[ n \right] = \overline{Y_1 \left[ n-1 \right]} \]

\[ Y_1 \left[ n \right] = Y_0 \left[ n-1 \right] \]
Problem 3 (40 Points):

Consider the current-mode circuit shown below.

1. (5 points) Express the currents through MOS transistors M1 through M4 in terms of input and output currents $I_{in}$, $I_{out}$ and supplied reference currents $I_a$ and $I_b$.

2. (35 points) Find the output current $I_{out}$ as a function of input current $I_{in}$ and reference currents $I_a$ and $I_b$, assuming the transistors are identically sized and operate in the subthreshold region. You may ignore the Early effect ($\lambda = 0$), and assume the output voltage $V_{out}$ is sufficiently large.

3. BONUS: (15 extra points) If $I_a$ represents a 'real' current source realized by an nMOS transistor with gate bias $V_{a'}$, indicate what will happen when the input current reaches lower than a given level of current. Quantify your answer, expressing the effect on the output current.

![Circuit Diagram]

1. $I_1 = I_{in}$
2. $I_3 = I_b$
3. $I_2 = I_a - I_3 = I_a - I_b$
4. $I_4 = I_{out}$
2. \( I_{in} = I_0^{'} \frac{W}{L} e^{\frac{KV_{in}}{V_t}} \)  \hspace{1cm} (M1)

\( I_{a-I_b} = I_0^{'} \frac{W}{L} e^{\frac{(KV_{in}-V_S)}{V_t}} \)  \hspace{1cm} (M2)

\( I_{b} = I_0^{'} \frac{W}{L} e^{\frac{(KV_{b}-V_S)}{V_t}} \)  \hspace{1cm} (M3)

\( I_{out} = I_0^{'} \frac{W}{L} e^{\frac{KV_b}{V_t}} \)  \hspace{1cm} (M4)

\[ \Rightarrow \] \[ \frac{(M4)}{(M3)} = 1 \]

\[ \frac{(M4)}{(M3)(M1)} = 1 \]

\[ \Rightarrow I_{out} = \frac{I_b}{I_{a-I_b}} \cdot I_{in} \]  \hspace{1cm} (5)

3. \[ \begin{array}{c}
\downarrow I_a \\
V_S
\end{array} \] \[ \downarrow I_{out} \]

\[ V_a \downarrow \frac{V_S}{L} \]

\[ I_{out} = \frac{I_0^{'}W}{L} e^{\frac{KV_a}{V_t}} \left(1 - e^{-\frac{V_S}{V_t}}\right) \]

\[ \frac{(M3)}{(M4)} \Rightarrow e^{-\frac{V_S}{V_t}} = \frac{I_b}{I_{out}} \Rightarrow I_a = I_a^{o} \left(1 - \frac{I_b}{I_{out}}\right) \]

\[ \text{in (5)} \Rightarrow I_{out} \left( I_a^{o} \left(1 - \frac{I_b}{I_{out}}\right) - I_b \right) = I_b \cdot I_{in} \]

\[ I_{out} \left( I_a^{o} - I_b \right) = I_b \cdot I_{in} + I_b \cdot I_a^{o} \]

\[ \Rightarrow I_{out} = \frac{I_b}{I_a^{o} - I_b} \cdot (I_{in} + I_a^{o}) \]

\[ \text{elim term (-I_{out})} \]